Philosophy 240
Symbolic Logic

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Fall 2010

Class 4: Conditionals

Natural-Language Conditionals

A. Indicative conditionals: If the Mets lost, then the Cubs won.
B. Conditional questions: If I like logic, what class should I take next?
C. Conditional commands: If you want to pass this class, do the homework.
D. Conditional prescriptions: If you want a good life, you ought to act virtuously.
E. Cookie Conditionals: There are cookies in the jar, if you want them.
F. Subjunctive conditionals: If Rod were offered the bribe, he would take it.

- B, C, and D are not propositions; as they stand, they lack truth values.
- We can parse them truth-functionally:

  B'. If you like logic, then you take linear algebra next.
  C'. If you want to pass the class, you do the homework.
  D'. If you want a good life, you act virtuously.
- E isn't really even a conditional statement; it's a total fraud.
- F is a worry.

The Material Conditional

α	⊃	β
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊤	⊤
⊥	⊤	⊥

1. Why this is weird.
2. Why we can't do much about the weirdness.

Or, more precisely,

   1. Is the material conditional the best truth-functional interpretation?
   2. Should we give up truth-functionality?

Logical Truths

- Privileged sentences of a logical system
- The theorems of a system of logic
- In an axiomatic system we choose a small set of privileged sentences that we call axioms.
  - Euclidean geometry
  - Newtonian mechanics
- Axioms be obvious and uncontroversial.
- The theorems of a formal system are the statements that are provable from the axioms.
- We identify any formal system with its theorems.
- In propositional logic, the theorems are also called tautologies.
  - true in every row of the truth table
  - $P \supset P$
  - $[(P \supset (Q \supset R)] \supset [(P \supset Q) \supset (P \supset R)]$
- "Demarcate the totality of logical truths, in whatever terms, and you have in those terms specified the logic" (Quine, *Philosophy of Logic*, p 80.)

Paradoxes of Material Implication

- Statements of the following forms are tautologies:

  $\alpha \supset (\beta \supset \alpha)$

  $\sim\alpha \supset (\alpha \supset \beta)$

  $(\alpha \supset \beta) \vee (\beta \supset \alpha)$

- Unobvious

- Awkward consequences

  'If Martians have infra-red vision, then Obama is president' is true.

  'If Bush is still president, then Venusians have a colony on the dark side of Mercury' is true.

  Either 'Neptunians love to wassail' entails 'Saturnians love to foxtrot' or 'Saturnians love to foxtrot' entails 'Neptunians love to wassail'.

  One of the following is true:

  – 'It is raining' implies 'Chickens are robot spies from Pluto'.

  – 'Chickens are robot spies from Pluto' implies 'It is raining'.

Dependent and Independent Conditionals

- A *dependent conditional* has a connection between its antecedent and consequent.
  - If it is raining, then I will get wet.
  - If I run a red light, then I break the law.
  - If the car is running, then it has fuel in the tank.
  - If I were to jump out of the window right now, I would fall to the ground.
- An *independent conditional* lacks the connection we find in a dependent conditional
  - If 2+2=4, then cats are animals.
  - If 2+2=4, then cats are robots.
  - If pigs fly, then Utica near Rome.
  - If pigs fly, then Utica is the capital of Canada.
- The material interpretation is awkward for independent conditionals.

Benefits of the Material Interpretation

- Truth-Functional Compositionality: the truth value of any complex sentence is completely dependent on the truth value of its component parts.
- Imagine you were using logic to program a computer, or a robot.
- We do not want the program to stall on an empty truth value.
- We want it to have rules for how to proceed in any case.
- The dependent conditional is more common than the independent one.
- We don’t have strong feelings about the truth values of independent conditionals.

Nicod's Criterion

- The first two lines of the material interpretation are fine.
- Many scientific laws are conditional in form.
- Nicod's criterion for confirmation of a scientific claim
  - Evidence confirms a law if it satisfies both the antecedent and consequent.
  - Evidence disconfirms a law if it satisfies the antecedent, and fails to satisfy the consequent.
- Coulomb's Law: $F = k\ |q_1q_2| / r^2$.
  - If two particles have a certain amount of charge and a certain distance between them, then they have a certain, calculable force between them.
  - If we were to find two particles which did not have the force between them that the formula on the right side of Coulomb's Law says should hold, and we could not find over-riding laws to explain this discrepancy, we would seek a revision of Coulomb's Law.
- All swans are white.
  - If something is a swan, then it is white.
  - When we find a white swan, which satisfies the antecedent and the consequent, it confirms the claim.
  - If we were to find a black swan, which satisfies the antecedent but falsifies the consequent, then it would disconfirm the claim.

Nicod’s Criterion and the Second Two Rows

- According to Nicod’s criterion, instances which do not satisfy the antecedent are irrelevant to confirmation or disconfirmation.
- A white dog and a black dog and a blue pen have no effect on our confidence in the claim that all swans are white.
- Call a conditional in which the antecedent is false a *counterfactual conditional*.
- Nicod’s criterion says nothing about counterfactual conditionals.
- We are considering alternatives to the material interpretation of the conditional.
- The point of mentioning Nicod’s criterion was to say that we should leave the first two lines of the truth table alone.

The Immutability of the Last Two Rows of the Truth Table for the Material Conditional

Option A

α	⊃	β
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊤	⊤
⊥	⊥	⊥

Option B

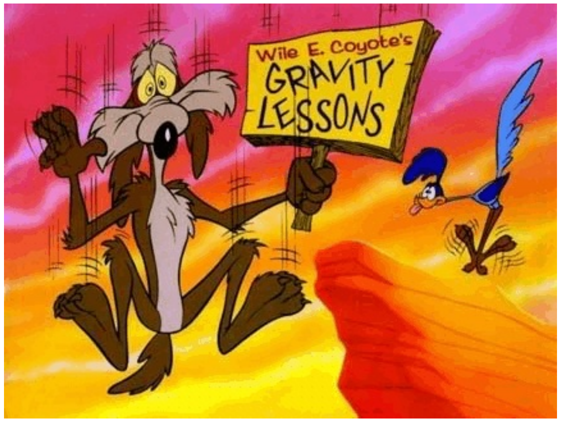
α	⊃	β
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊥	⊤
⊥	⊤	⊥

Option C

α	⊃	β
⊤	⊤	⊤
⊤	⊥	⊥
⊥	⊥	⊤
⊥	⊥	⊥

- Option A gives the conditional the same truth-values as the consequent.
- Option B gives the conditional the same truth-values as a biconditional.
- Option C gives the conditional the same truth-values as the conjunction.
- Thus, the truth table for the material conditional is the only one possible with those first two lines that doesn't merely replicate a truth table we already have.

Counterfactual Dependent Conditionals



- If I were to jump out of the window right now, I would fall to the ground.
  - Option A says that this sentence is falsified when I don't jump out the window and I don't fall to the ground.
  - Options B and C say that it is falsified when I don't jump out of the window and I do fall to the ground.
  - But, neither case seems to falsify the sentence, as it is intended.
- The only time that sentence is falsified, as on Nicod's criterion, is in the second line of the truth table.
- We must stick with the original truth table, if we want the conditional to be truth-functional.
- If the problem were just the oddities of the paradoxes of the material conditional and independent conditionals, we might reast easily biting the bullet.
- But, the problem is deeper.

Subjunctive and Counterfactual Conditionals

- If Rod were offered the bribe, he would take it.
  - If Rod takes the bribe, then it is true; if he refuses the bribe, then it is false.
  - If Rod is never offered the bribe, then it is true.
- But contrast:
  - S: If I were to jump out of the window right now, I would fall to the ground.
  - S': If I were to jump out of the window right now, I would flutter to the moon.
- I am not now jumping out of the window.
- But, S is true and S' is false.
- Some conditionals with false antecedents are false!
- Goodman’s Example
  - If that piece of butter had been heated to 150°F, it would not have melted.

Non-Truth-Functional Operators

The Two-State Solution

- The logical aspects of the natural-language conditional may be expressed by the truth-functional '⊃' .
- Other aspects of the natural-language conditional might not be truth-functional.
- We could introduce a new operator, strict implication, ⇒.
- Statements of the form 'α ⊃ β' could continue to be truth-functional
- Statements of the form 'α ⇒ β' would be non-truth-functional.
- If 2+2=4, then cats are animals.
  - We could regiment it as 'T ⊃ C' in the standard way.
  - Or we could regiment it as 'T ⇒ C'.
  - 'T ⇒ C' would lack a standard truth-value in the third and fourth rows.
- Cookie conditionals

Modal Interpretations

- C.I. Lewis defined ‘$\alpha \Rightarrow \beta$’ as ‘$\Box(\alpha \supset \beta)$’.
- The ‘$\Box$’ is a modal operator.
- Modal operators can be used to construct formal theories of knowledge, moral properties, tenses, or knowledge.
- For Lewis’s suggestion, strict implication, we use an alethic interpretation of the modal operator.
  - ‘$\Box$’ means ‘necessarily’.
- On the modal interpretation ‘$\alpha \Rightarrow \beta$’ will be true if it is necessarily the case that the consequent is true whenever the antecedent is.
  - If I were to jump out of the window right now, I would fall to the ground.
  - If I were to jump out of the window right now, I would flutter to the moon.
  - If 2+2=4, then cats are animals.
  - If that piece of butter had been heated to 150°F, it would not have melted.
- Modal logics, while currently popular, are controversial.
  - Necessity and contingency may not be properly logical topics.
  - Our ability to know which events or properties are necessary and which are contingent is severely limited.

Modal Operators and Laws

- One advantage of introducing a modal operator to express implication is that it connects conditional statements and scientific laws.
- A scientific law is naturally taken as describing a necessary, causal relation.
- When we say that event A causes event B, we imply that A necessitates B, that B could not fail to occur, given A.
- To say that lighting the stove causes the water to boil is to say that, given the stability of background conditions, the water has no choice but to boil.
- Thus, we might distinguish the two senses of the conditional by saying that material implication represents logical connections, where strict implication attempts to regiment causal connections.

Counterfactual Conditionals and Causal Laws

- The natural-language conditional often indicates a connection between its antecedent and its consequent.
    1. If I run a red light, then I break the law.
    2. If you paint my house, I will give you five thousand dollars.
    3. If it is raining, then I will get wet.
    4. If the car is running, then it has fuel in the tank.
    5. If I were to jump out of the window right now, I would fall to the ground.
- The connection in 1 is mainly conventional.
- 2 refers to an offer or promise.
- But, 3-5 are fundamentally causal, depending on basic scientific laws.

Causation and Conditionals

- Causal laws are often conditional, indicating dispositional properties.
- 'If this salt had been placed in water, it would have dissolved.'
  - indicates a dispositional property of salt
- Other dispositional properties, like irritability, flammability, and flexibility, refer to properties interesting to scientists.
- Psychological properties, are often explained as dispositions to behave.
  - Believing that it is cold outside
- This marble counter is soluble in water.
  - If we never place the counter in water, then it comes out true on the material interpretation.
- To be flammable is just, by definition, to have certain counterfactual properties.
- These pajamas are flammable just in case they would burn if subjected to certain conditions.
- The laws of science depend essentially on precisely the counterfactual conditionals that the logic of the material conditional gets wrong.

Goodman, Counterfactuals, and Laws

- Goodman: Counterfactual conditionals are not merely logical relations.
- The problem of giving an analysis of the logic of conditionals is intimately related to the problem of distinguishing laws from accidental generalizations.
- Compare:
  1. There are no balls of uranium one mile in diameter.
  2. There are no balls of gold one mile in diameter.
- The explanation of 1 refers to scientific laws about critical mass.
- The explanation of 2 is merely accidental.
- In order to know that difference, you must know the laws which govern the universe.
- The problem of knowing the laws of nature is inextricably linked to the problem of understanding the logic of the natural-language conditional.

Background Conditions

- We sometimes use conditionals as truth-functional connectives.
- More commonly, especially in counterfactual cases, we use them to state connections between antecedents and consequents.
- A conditional will be true if the relevant connections hold among the antecedent and the consequent, and false if such connections do not hold.
  - "A counterfactual is true if a certain connection obtains between the antecedent and the consequent. But...the consequent seldom follows from the antecedent by logic alone" (7-8).
- Consider: If that match had been scratched, it would have lighted.
- For the claim to be true, there have to be a host of other conditions satisfied.
  - "We mean that conditions are such - i.e. the match is well made, is dry enough, oxygen enough is present, etc. - that "That match lights" can be inferred from "That match is scratched." Thus the connection we affirm may be regarded as joining the consequent with the conjunction of the antecedent *and other statements that truly describe relevant conditions*" (8-9, emphasis added).

Relevance

- When we assert a conditional, we commit ourselves to the relevant related claims.
- But, to understand the claims to which we are committed, we must understand the relevant connections between the antecedent and the consequent.
- We must understand the general laws connecting them.

  "The principle that permits inference of 'That match lights' from 'That match is scratched. That match is dry enough. Enough oxygen is present. Etc.' is not a law of logic but what we call a natural or physical or causal law" (8-9).

Summing Up

- The proper analysis of counterfactual conditionals is not a logical matter.
- Goodman claims that we need two elements:
  1. A definition of the conditions that are relevant to the inference of a consequent from an antecedent.
  2. A characterization of causal laws.
- We have gone far from just understanding the logic of our language.
- We are now engaged in a pursuit of the most fundamental features of scientific discourse.