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Derivations Using Identity I (§8.7)

I. The three ID rules

We saw that there are three rules governing identity (ID).

1. Reflexivity: α=α
2. Symmetry: α=β :: β=α
3. Indiscernibility of Identicals: $\mathscr{F}$α
   α=β / $\mathscr{F}$β

Reflexivity is an axiom schema.
Symmetry and indiscernibility are rules of replacement.
Thus, we use them differently.
We can add an instance of the axiom schema into any proof, with no line justification.
We can use symmetry on whole lines or on parts of lines.
With indiscernibility, we are always re-writing a whole line, switching one constant for another.

II. Derivations in identity theory

Consider the original problem from when we started identity theory.

Superman can fly.
Superman is Clark Kent.
∴ Clark Kent can fly.

1. Fs	
2. s=c	/ Fc
3. Fc	1, 2, ID

QED

Using the symmetry rule:

1. a=b ⊃ j=k	
2. b=a	
3. Fj	/ Fk
4. a=b	2, Id
5. j=k	1, 4, MP
6. Fk	3, 5, Id

QED

To derive the negation of an identity statement, one often uses IP:

```
1. Rm
2. ~Rj              / m≠j
        |3. m=j
        |4. Rj
        |5. Rj · ~Rj
6. m≠j
QED
```

Using the reflexivity rule:

```
1. (x)(~Gx ⊃ x≠d)        / Gd
        |2. ~Gd              AIP
        |3. ~Gd ⊃ d≠d        1, UI
        |4. d=d              ID
        |5. d≠d              3, 2, MP
        |6. d=d · d≠d        4, 5, Conj
7. Gd
QED
```

An existential conclusion:

```
1. Rab
2. (∃x)~Rxb              /(∃x)~x=a
3. ~Rcb                      2, EI
        |4. c=a              AIP
        |5. Rcb              1, ID
        |6. Rcb · ~Rcb       5, 3, Conj
7. ~c=a                      4-6, IP
8. (∃x)~x=a                  7, EG
QED
```

Translate and derive:

The Joyce scholar at Hamilton is erudite. Therefore, all Joyce scholars at Hamilton are erudite.

(∃x){(Jx · Hx) · (y)[(Jy · Hy) ⊃ x=y] · Ex}   / (x)[(Jx · Hx) ⊃ Ex]

Note that I have dropped one set of brackets in the premise.
Again, at this point in the term, you may drop brackets from series of conjunctions or disjunctions.

The argument may seem a little odd, since it derives a universal conclusion from an existential premise.
Remember that a definite description is definite; there is only one thing that fits the description.
The universality of the conclusion is supported by the uniqueness clause in the definite description.

1. (∃x){(Jx · Hx) · (y)[(Jy · Hy) ⊃ x=y] · Ex}	/ (x)[(Jx · Hx) ⊃ Ex]
2. ~(x)[(Jx · Hx) ⊃ Ex]	AIP
3. (∃x)~[(Jx · Hx) ⊃ Ex]	2, CQ
4. ~[(Ja · Ha) ⊃ Ea]	3, EI
5. ~[~(Ja · Ha) ∨ Ea]	4, Impl
6. (Ja · Ha) · ~Ea	5, DM, DN
7. (Jb · Hb) · (y)[(Jy · Hy) ⊃ b=y] · Eb	1, EI (to b)
8. (y)[(Jy · Hy) ⊃ b=y]	7, Com, Simp
9. (Ja · Ha) ⊃ b=a	8, UI (to a)
10. Ja · Ha	6, Simp
11. b=a	9, 10, MP
12. Eb	7, Simp
13. Ea	12, 11, ID
14. ~Ea	6, Com, Simp
15. Ea · ~Ea	13, 14, Conj
16. (x)[(Jx · Hx) ⊃ Ex]	2-15, IP

QED

III. Exercises. Derive the conclusions of each of the following arguments.

1.
1. (x)(Dx ⊃ Ex)
2. Da
3. a=b / Eb

2.
1. (x)(Ax ⊃ Bx)
2. ~Bf
3. Ae / f≠e

3.
1. (x)(Hx ⊃ Jx)
2. (x)(Kx ⊃ Lx)
3. Hd · Kc
4. c=d / Jc · Ld

4.
1. (x)(y)x=y
2. (x)Mxx / Mab

5.
1. (x)[(∃y)Kxy ⊃ (∃z)Kzx]
2. (∃x)(Kxg · x=b) / (∃z)Kzb