Class 36 - November 19
Philosophy Friday #7: The Color Incompatibility Problem
Katz, "The Problem in Twentieth-Century Philosophy"

I. Wittgenstein's Logical Atomism

The success of Frege's mathematical logic was so great that some philosophers supported hopes of it being the canonical language of all of knowledge.
Through the twentieth century, Quine was a proponent of that view, as we saw in his chapter on deviant logics.
Some philosophers continue to believe that first-order logic is canonical.
But, the original philosophical application of Frege's logic to broader philosophical purposes came from Ludwig Wittgenstein, in his 1919 *Tractatus Logico-Philosophicus*.
Wittgenstein sent the *Tractatus* to Bertrand Russell in Cambridge on scraps of paper from the trenches in WWI, where Wittgenstein was in the Austrian army.
Wittgenstein's *Tractatus* was intended, and was hailed by Russell, as the culmination of the enterprise of logical analysis begun by Frege.
According to the *Tractatus,* the world is a collection of independent atomic facts combined according to logical principles.
If we could get clear about the correct logic, Wittgenstein argued, then we could have a complete, accurate picture of the world in our best, most austere language.

The *Tractatus* was highly influential in Europe between the two wars, as the foundation of logical empiricism, or logical positivism.
A group of positivist philosophers influenced by the *Tractatus*, including Rudolph Carnap, Otto Neurath, Moritz Schlick, and Herbert Feigl, came to be known as the Vienna Circle.
A similar, though less-influential, group called the Berlin Circle was centered around the physicist Hans Reichenbach.
The young A.J. Ayer visited Vienna from England and wrote about the movement he found there.
Ayer's *Language, Truth, and Logic* became the primary source for positivism for English-speaking philosophers, though most of the positivist's central works eventually were translated into English.

We could easily spend an entire term studying the *Tractatus*, let alone logical positivism.
The Tractatus is obscure, when read directly, consisting of a series of numbered aphorisms.
There are seven main propositions, and all but the seventh have sets of explanatory sub-propositions.
Wittgenstein seeks the limits of language in distinguishing between what can and what can not be said.

    §7. Whereof one cannot speak, thereof one must be silent.

The project of distinguishing between what can and can not be said, or between what can and can not be thought, naturally meets a fundamental difficulty.
If we want to distinguish between the backyards of two people, we can draw a boundary line.
We perceive both sides of the line, and see the landscape divided.
But, what is outside of the boundary of thought is inaccessible to us.
Still, we can at least try to get clear about how our language functions, and what its limits are.

The *Tractatus* presents an atomistic picture theory of meaning on which language mirrors the world.
The structure both of language and of the world is governed by logical rules, like those depicted in the truth tables.
Indeed, Wittgenstein was the first to develop truth tables, in the *Tractatus*; see §4.31.
The world, he alleges, is a collection of independent states of affairs.
If I am standing to the right of you, we have, let's say, two atomic facts (my standing and your standing) and a logical relation (to the right of) between those facts.
I could stand to the right of you, or to the left of you, or on the other side of the planet, all of which are independent of you.

> §1.2. The world divides into facts.
> §2.06. From the existence or non-existence of one state of affairs, it is impossible to infer the existence or non-existence of another.

Language consists of atomic statements of those facts, connected into more complex statements by logical principles.
Language mirrors the world by providing a logical structure which is isomorphic to the structure of the world.

> §2.16. If a fact is to be a picture, it must have something in common with what it depicts.
> §2.17. What a picture must have in common with reality, in order to be able to depict it - correctly or incorrectly - in the way it does, is its pictorial form.

Since language and logic have the same form as the world, we can know about the fundamental structure of reality by examining the fundamental structures of language and logic.

Of course, we can not rely on the surface grammar of natural language to reflect the structure of the world.
As we have seen throughout this course, natural language is sloppy full of misleading metaphors and pragmatic shorthand.
If we want a true representation of the world, we must seek a finer language, like Frege's mathematical logic.
Recall Frege's claim that his *Begriffsschrift* is like a microscope on our language.

> I believe I can make the relationship of my *Begriffsschrift* to ordinary language clearest if I compare it to that of the microscope to the eye. The latter, due to the range of its applicability, due to the flexibility with which it is able to adapt to the most diverse circumstances, has a great superiority over the microscope. Considered as an optical instrument, it admittedly reveals many imperfections, which usually remain unnoticed only because of its intimate connection with mental life. But as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate. The microscope, on the other hand, is perfectly suited for such purposes... (Frege, Preface to *Begriffsschrift*)

Frege's logic, therefore, is the precision tool that Wittgenstein's picture theory requires to represent the atomic facts of the world, and to show how they are related and combined.
To see how the demands for precision are manifested, notice that my example of an atomic fact, my standing to the right of you, is misleading.
My standing in a place is not an atomic fact, it is a complex fact.

I am a complex, standing is a complex, you are a complex.
The true analysis of the world involves analyzing these complexes into their simple, atomic components.
Atomic facts are the foundational elements for the *Tractatus*, akin to the postulates of Euclidean geometry, say, or to Descartes's *cogito*.
A theory of the world that analyzed all of the myriad complexes into their atomic elements would present a veridical and secure picture of the world.
Because of its method of analyzing complex propositions into elementary ones, the kind of philosophy that was developed by the early Wittgenstein, under the influence of Frege and Russell, was thus called analytic philosophy.
The name 'analytic philosophy' remains as a characterization of Anglo-American philosophy, despite the lack of contemporary interest in the project of analysis, in this sense.
But Wittgenstein's original plan was to use the new logic, because of its utility for analysis, to represent the atomic facts fo the world in elementary propositions and their logical combinations.

II. The Problem

Wittgenstein never gives a clear example of an atomic fact.
Russell used the example of the color of a spot in my field of vision.
A spot in one's field of vision seems as likely a candidate for an atomic fact as any.
Atomic facts were supposed, by definition, to be all independent.
Wittgenstein noticed that even such facts carry some sort of entailment relations.

> §6.3751. It is clear that the logical product of two elementary propositions can neither be a tautology nor a contradiction. The statement that a point in the visual field has two different colors at the same time is a contradiction.

The atomic facts both must be, and are not, independent.
Jerrold Katz characterizes Wittgenstein's 6.3751 as the central problem in twentieth-century philosophy.
To explicate the problem, he considers:

> 1. The spot is red and blue.
> 2. The spot is red.
> 3. The spot is not blue.
> 4. The spot has a color.
> 5. Red is a color.
> 6. The spot is green.

Each of the propositions used in 1-6 is supposed to represent an atomic fact; 1 and 3 are logical products of simpler elementary propositions.
But 1 is a contradiction.
2 and 5 are incompatible, and 2 entails 3 and 4.
There are some kinds of logical relations among these propositions, even though they appear to be elementary.
The world appears not to be atomic, in the way that the *Tractatus* depicts.
If the elementary propositions are inter-dependent, it is difficult to see how they could server as the foundations of other beliefs.

If the proposition that this spot is green entails that it is not red, and not purple, and that it is a color, and that spots are incompatible with each other, and so on, I can not just immediately and securely know a single, simple fact.

The problem of how to understand how elementary propositions can have logical relations among them has become known as the color incompatibility, or color exclusion, problem.
As Katz observes, the problem is not merely about color.

> It is a general problem about the extralogical vocabulary of the language and about all the semantic properties and relations of the language (548).

The problem can be seen in any sentence whose truth seems to be both logical, and depending on the meanings of terms.
7 appears to be a special kind of sentence, one whose truth is guaranteed by its meaning, like a logical truth.

> 7. Bachelors are single.

We can regiment 7 into predicate logic.

> 7'. $(x)(Bx \supset Sx)$

But, the logic does not reveal the special status of 7.
There are some sorts of logical relations among the terms 'bachelor' and 'single'.
But, the logic we have been studying does not show those relations.

III. Meaning Postulates

It might seem rather easy to treat the color incompatibility problem.
We can just adopt statements like 8-10 as axioms.

> 8. All bachelors are unmarried.
> 9. Red is a color.
> 10. Red is not blue.

This proposal was explored by Carnap, in his paper "Meaning Postulates."
Propositions like 8-10 are, as Katz notes, extra-logical; they are about meanings rather than about logic.
Carnap's proposal is that we can stipulate whatever meaning relations we think are important.

The stipulation involved in adopting meaning postulates leads to two serious problems.
First, we would have to adopt a lot of meaning postulates.
Red is not blue, and not green, and not a ball o0f feta cheese, and not the Archbishop of Canterbury.
It is not plausible that we believe all of the required meaning postulates.

Second, and more problematically, a long list of meaning postulates is not an explanation of why these postulates hold.

Meaning postulates serve as constraints on the assignment of extensions to sentences, but they cannot explain the property common to the sentences they enumerate. Like Socrates's interlocutors, meaning postulates offer examples of the concept instead of the concept (Katz 553).

This second problem with meaning postulates is subtle, so let's take a moment to spell it out carefully. If I stipulate that all Ps or Qs are Rs, then it will follow that all Ps are Rs.

11. $(x)[(Px \lor Qx) \supset Rx)] \supset (x)(Px \supset Rx)$

11 is a logical truth.
But, 11 holds for any values of P, Q, and R.
11 does not tell us anything about the relationship between Ps and Rs.
It does not tell us that there is a relationship between being a P that entails being an R.
It says, for example, that if all tall bachelors are single, then all bachelors are single.
But, we want an explanation of the relationship between being a bachelor and being single.
We want an explanation of the consequent of 11, not merely that it follows from its antecedent as a logical truth.

Here's another way to look at the problem.
I can stipulate anything I want.
So, I can stipulate that all Beethoven symphonies are boring.
Given that stipulation, it will follow as a matter of logic that the *Eroica* (Beethoven's third symphony) is boring.
But, it is not a logical truth that the *Eroica* is boring.
We want the fact that this spot is not blue to follow as a matter of logic from the fact that it is homogeneously red.

Compare 11 with 12, which is a logical truth.

12. $(P \supset Q) \supset [(Q \supset R) \supset (P \supset R)]$

12 is not the result of any stipulation.
It is a theorem of our logic.
If meaning postulates were able to do the work that Carnap wants them to do, they would give the status that 12 has to the consequent of 11.
Using meaning postulates to solve the color incompatibility problem makes sentences 1-6 true by stipulation.
But we can stipulate anything we like.
We want the truth of propositions 1-6 to be true as a matter of the logic of the terms, like 12, rather than as a matter of stipulation, like 11.

IV. Semantic Markers

In order to avoid the problems with Carnapian meaning postulates, Katz proposes a constraint on any solution of the color incompatibility problem.

> A new way out must reject Carnap's assumption that the external, logical structure of extralogical words is the source of analyticity, contradiction, and analytic entailment in connection with sentences like (1)-(6). It must assume instead that such properties and relations derive from the internal, sense structure of extralogical words (Katz 553).

Katz proposes that in addition to the mathematical logic of Frege, we need a formal theory of semantic entailment, one that gets to the analyticity of meanings.
Just as we went beneath the level of the sentence moving from **PL** to **M**, we can move beneath the level of logical form to semantic form.

Katz calls the semantic structural properties of syntactically simple terms decompositional sense structure.
Senses are meanings.
Decompositional sense structure is not syntactic.
It depends essentially on meanings, and not the form of terms.
A term like 'bachelor', which is syntactically simple, can be semantically complex.

> The sense of 'single man' is complex, being a compositional function of the senses of 'single' and 'man'. Since 'single man' and 'bachelor' have the same sense, the sense of 'bachelor' is complex (Katz 555).

Also, decompositional sense structure is not logical, as the color incompatibility problem shows; the consequent of 11 is nothing like a logical truth.
I will not pursue the details of decompositional sense structure, which you can see in §IV- §VI of Katz's paper.
The central point is that one can analyze the concept of being a color in such a way as to reveal the semantic entailments like the ones expressed in 1-6.
Katz uses semantic markers to represent the decompositional sense structure of what appeared to Wittgenstein to be elementary propositions
'This spot is blue' is not a semantically elementary proposition; it presupposes a variety of analytic entailments.

On Katz's analysis, blueness can still be a primitive sense, in that it is not definable in terms of other senses.
But, the primitiveness of the sense does not entail that it is semantically simple.
It has analytic relations with other senses, despite being primitive.
Katz calls the senses of basic color terms complex primitive senses.
They are primitive in that they are not reducible to other senses.
They are complex, since they have semantic relations to other senses.

Senses are thus both inside and outside of logic.
Sense entailments are additional to logical ones.
But, they constrain logic, since they guide entailments.

> Since senses provide the fine-grained linguistic structure necessary for a model-theoretic explanation of why such sentences have such logical properties and relations, senses are inside logic in precisely Wittgenstein's sense of "hav[ing] an effect on one proposition's following from another" (Katz 572).

Katz's semantic markers have not caught on in the philosophical community at large.
While they are patterned after Noam Chomsky's syntactic theories of language, they are much more contentious.
Many philosophers are wary of meanings.
Senses are objective, in that they transcend any particular thinker or language user.
But they are not the kinds of things that we can perceive with our senses.
Thus, some philosophers think of them as spooky entities.
Still, they give us a way of understanding the semantic relations among terms without abandoning Wittgenstein's atomism.
The more popular response to Wittgenstein's problem is holistic, abandoning the atomism of the positivists.
Many of the more prominent holists, like Quine, also deny the existence of meanings.

V. Positivism and Holism

Color incompatibility is a puzzle for both Wittgensteinian atomists and the positivists that followed Wittgenstein because it looks as if there is a logical relationship between various atomic facts.
To see how the problem manifests itself for positivism, we need to look more closely at the broader aims of that philosophical movement.

The positivists saw Wittgenstein's picture theory as accommodating a scientific view of the world.
Scientific laws, for example, were mere generalization over, and reducible to, the separable atomic facts.
The positivists believed that all our legitimate claims are traceable to a core set of simple observations.

> There is a class of empirical propositions of which it is permissible to say that they can be verified conclusively. It is characteristic of these propositions, which I have elsewhere called "basic propositions," that they refer solely to the content of a single experience, and what may be said to verify them conclusively is the occurrence of the experience to which they uniquely refer...Propositions of this kind are "incorrigible,"...[in that] it is impossible to be mistaken about them except in a verbal sense (Ayer, *Language Truth and Logic*, p 10).

The positivists claimed that all of science and philosophy could be founded on the basis of observation statements in conjunction with the logical and mathematical principles used to regiment and derive those observations.
Claims that are not observable may be derived from axiomatic observations, or introduced by definition.
Lastly, some claims, like logical truths, are neither observable nor derivable from observable claims.
Hume called such claims relations of ideas.
The positivists called them analytic truths.
Among the analytic truths were supposed to be logical truths and, for some philosophers like Frege, the truths of mathematics.
For the positivists, all and only meaningful statements will be either analytic, observable, or derivable (using logic) from observable axioms.

A fundamental presupposition of positivism, then, is that one can make a clear distinction between an observation statement and an analytic one.
This distinction was rooted in Wittgenstein's distinction between sensible statements and logical nonsense.

One of the most important advances in modern logic was its ability to characterize logical truth.
We might characterize logical truths as necessary truths.
For Descartes, the certainty of logic and mathematics had provided essential support to his claim that our minds have substantial content built into their structures.
From the claim that logic and mathematics are innate, it is reasonable to ask whether there are other innate ideas, including the idea of God.
Wittgenstein thought that characterizing logical truths as necessary imbues them with too much importance.
In contrast, he called them nonsense.
The only statements that can picture the world are those that have sense, that can be either true or false, that can picture accurately or not.
Tautologies are empty of content.

> §4.46. The proposition shows what it says, the tautology and the contradiction that they say nothing. The tautology has no truth conditions, for it is unconditionally true; and the contradiction is on no condition true. Tautology and contradiction are without sense.
> §6.1251. Hence, there can *never* be surprises in logic.

Logical truths are unknowable because they are too thin to be objects of knowledge.
They don't picture any fact.
Wittgenstein wanted carefully to circumscribe what we can know.

The logical truths were for Wittgenstein logical nonsense, and for the positivists, merely analytic.
All agreed, though, that they were easily derivable within formal logic.
And the analytic truths were sharply in contrast with the synthetic ones, which had to trace back, in some way, to observation.
Indeed, the whole of the atomist movement, from Locke and Hume through Wittgenstein and the positivists seemed to rest on this distinction between analytic and synthetic propositions.

Quine attacked the positivist's distinction between analytic and synthetic statements by arguing for holism.

> It is obvious that truth in general depends on both language and extralinguistic fact. The statement "Brutus killed Caesar" would be false if the world had been different in certain ways, but it would also be false if the word "killed" happened rather to have the sense of "begat." Hence, the temptation to suppose in general that the truth of a statement is somehow analyzable into a linguistic component and a factual component. Given this supposition, it next seems reasonable that in some statements the factual component should be null; and these are the analytic statements. But, for all its a priori reasonableness, a boundary between analytic and synthetic statements simply has not been drawn (Quine, "Two Dogmas of Empiricism," 70).

Our knowledge of synthetic propositions are supposed to be rooted in sense experience of particular facts.
But the particular beliefs that are supposed to be the starting points of our knowledge seem not to be independent, as the color incompatibility problem shows.
They seem to require, or presuppose, a whole battery of other facts that come along with them.
To know that this spot is green entails knowledge that green is a color, that this spot is not red, and so on.
This problem seems to undermine the claim that any atomic fact is given, as a foundational belief.

If the basic facts are interconnected, they could not possibly be immediately perceivable.
They would only be comprehensible en masse.

> Our statements about the external world face the tribunal of sense experience not individually but only as a corporate body ("Two Dogmas of Empiricism" 41).

This problem with the analytic/synthetic distinction is connected to the inter-connectedness of individual statements, their involvement with a broader theory, in contrast to Wittgenstein's atomism.
Carl Hempel applied this problem to his account of scientific reasoning.

> In the language of science, and for similar reasons even in prescientific discourse, a single statement usually has no experiential implications. A single sentence in a scientific theory does not, as a rule, entail any observations sentences; consequences asserting the occurrence of certain observable phenomena can be derived from it only by conjoining it with a set of other, subsidiary, hypotheses (Hempel, "Empiricist Criteria of Cognitive Significance: Problems and Changes," 56).

Wittgenstein and the positivists presented a system on which individual sentences, pictures of states of affairs, were verified or not, and connected only by logic into a big theory.
The holist's claim is that the meaning of a single expression is elliptical, incomplete on its own.
It requires, for its meaning, reference to an entire linguistic framework, a theoretical context which forms the background to that expression.

> If...cognitive significance can be attributed to anything, then only to entire theoretical systems formulated in a language with a well-determined structure (Hempel, "Empiricist Criteria of Cognitive Significance: Problems and Changes," 57).

Hempel here alludes to what has come to be known as semantic holism: the unit of empirical significance is not the individual sentence, but the entire theory.

Holism comes in a variety of forms.
The strongest form, semantic holism claims that the meaning of any term or sentence depends on the meanings of all of our sentences.
Meaning is a property of an entire language, not of individual terms.
Less contentiously, confirmation holism claims that individual sentences are confirmed or refuted only by whole theories, not individually.
Confirmation holism is just a logical fact about sets of sentences.
Even two contradictory sentences are compatible in the absence of a larger theory which prohibits contradiction.
Wilfrid Sellars argues that the holistic conclusion is not merely about colors, and observation reports of them.

> It follows, as a matter of simple logic, that one couldn't have observational knowledge of *any* fact unless one knew many *other* things as well (Sellars, "Does Empirical Knowledge Have a Foundation?" 123).

If holism, even in its weak form, is correct, then the presupposition of atomism that some of our beliefs can serve as unassailable foundations for the rest is false.

Holist criticisms undermine any given-ness of our purportedly basic beliefs.

Given the constraints on knowledge, we could not know any particular fact unless we already knew a broader swath of background facts.
We could not know that a spot is green unless we already knew that green is a color, that a spot which is green is not red, and so on.

> One couldn't form the concept of *being green*, and, by parity of reasoning, of the other colors, unless he already had them (Sellars, "Does Empirical Knowledge Have a Foundation?" 120).

If knowing that this spot is green requires prior knowledge of a larger background theory, it becomes difficult to see how one could come to know anything at all.
The holist, then, has a strong critical argument against the atomist, but creates what seems to be an even more intractable problem.

VI. Summary

We have looked at two different kinds of responses to the color incompatibility problem.
Carnap and Katz attempt to save atomism by exploring the logic of semantic entailments.
Given first-order logic, there is no formal representation of the connections among 1-6.
But, we can extend our logic so that there is a formal representation of those entailments.

In contrast, holists like Quine, Sellars, and Hempel give up the belief that there are elementary propositions.
Quine, indeed, gives up on the idea that there are senses.
Quine denies that there are any logical connections among 1-6.
Instead, he believes that the connections are loose, at best causal connections.

These topics are far too broad to be considered in proper depth, here.
We have reached the edge of logic, and breached the barrier to the philosophy of language.

Paper Topics

1. The positivists were epistemic foundationalists, seeking to explain all of human knowledge on the basis of some secure, fundamental beliefs. Some critics of foundationalism, inspired by Quinean holism, defend coherentism in epistemology. Compare the two kinds of epistemologies. Sosa, Sellars, Ayer and Quine would all be good readings.

2. In "Two Dogmas of Empiricism," Quine argues against the positivist's reductionism. Evaluate Wittgenstein's project in light of Quine's criticisms. See Melchert for a good discussion of the *Tractatus*'s project, as well as Ayer.

3. Do meaning postulates solve the color incompatibility problem? See Carnap's article, as well as Quine's response in "Two Dogmas of Empiricism."

4. What are semantic markers? How do they attempt to solve the color incompatibility problem? In addition to the discussion in "The Problem...," see Katz's *Semantic Theory*.

5. How does the color incompatibility problem shift Wittgenstein away from his original project. Work through his "Remarks on Logical Form." See Allaire and/or Austin, as well.

6. What is the logical form of a sentence? Are there solutions, other than Carnap's, to the color incompatibility problem that rely on logical form? See the Pietroski article.

Suggested Readings

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