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Class 32 - November 10
Translation Using Relational Predicates I (§8.6)

I. Introducing Relational Predicates

Consider:

> Bob is taller than Charles.
> Andrew is taller than Bob.
> For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.
> So, Andrew is taller than Charles.

The conclusion should follow logically, but how do we translate the predicates?
If we only have monadic (1-place) predicates, like the ones we have so far considered, we have to translate the two first sentences with two different predicates:

> Bob is taller than Charles: Tb
> Andrew is taller than Bob: Ya

We really want a predicate that takes two objects. This is called a dyadic predicate. For examples:

> Txy: x is taller than y
> Kxy: x knows y
> Bxy: x believes y
> Dxy: x does y

We can have three-place predicates too, called triadic predicates:

> Gxyz: x gives y to z
> Kxyz: x kisses y in z
> Bxyz: x is between y and z

We can construct four-place and higher-place predicates.
All predicates which take more than one object are called relational, or polyadic.
We have a choice as to how to regiment relations.
Consider:

> Andrés loves Beatriz'

We could regiment it in monadic predicate logic:

> La

In that case, 'a' stands for Andrés, and 'L' stands for the property of loving Beatriz.
But, if we want to use 'L' to stand for the property of loving, it will have to take two objects: the lover and the lovee.

We can introduce a relational predicate, 'Lxy', which means that x loves y.
Then, we regiment the sentence:

Lab

Similarly, consider:

Camila gave David the earring.

We could use a predicate 'Gx', meaning that x gives David the earring.
But it is better to use a predicate 'Gxyz', meaning that x gives y to z.
Then, the sentence becomes:

Gcde

By using a relational predicate, we reveal more logical structure.
The more logical structure we reveal, the more we can facilitate inferences.

By introducing relational predicates, we have extended our language.
We are now using a language I called **F**, for Full First-Order Predicate Logic, rather than **M**.
For the purposes of this course, the differences between **F** and **M** are minor.
The only significant difference in the formation rules is in the construction of atomic formulas, at step 1.
But, beyond this course, the differences between **M** and **F** are significant; we have breached a barrier.
M admits of a decision procedure.
If a theory admits of a decision procedure, there is a way of deciding, for any given formula, whether it is a theorem or not.
F is not decidable.
There are formulas for which there are no effective methods for deciding whether they are theorems or not.

II. Relations: Syntax and Semantics

The shift from **M** to **F** requires just a small adjustment to the first of the syntactic rules.

Formation rules for wffs of F

1. An n-place predicate followed by n terms is a wff.
2. If α is a wff, so are
   $(\exists x)\alpha$, $(\exists y)\alpha$, $(\exists z)\alpha$, $(\exists w)\alpha$, $(\exists v)\alpha$
   $(x)\alpha$, $(y)\alpha$, $(z)\alpha$, $(w)\alpha$, $(v)\alpha$
3. If α is a wff, so is $\sim\alpha$.
4. If α and β are wffs, then so are:
   $(\alpha \cdot \beta)$
   $(\alpha \vee \beta)$
   $(\alpha \supset \beta)$
   $(\alpha \equiv \beta)$
5. These are the only ways to make wffs.

Remember that terms, for now, are either constants or variables; later we will add functions.
You can determine the value of 'n' in an n-place predicate just by counting the number of terms that follow the predicate letter.

The details of the semantics also have to be adjusted to account for relational predicates.
Recall that there were four steps for providing a standard formal semantics for **M**.

> **Step 1**. Specify a set to serve as a domain of interpretation, or domain of quantification.
> **Step 2**. Assign a member of the domain to each constant.
> **Step 3**. Assign some set of objects in the domain to each predicate.
> **Step 4**. Use the customary truth tables for the interpretation of the connectives.

The introduction of relational predicates requires adjustment to Step 3.
For an interpretation of **F**, we assign sets of ordered n-tuples to each relational predicate.

An n-tuple is an n-place relation.
Essentially, it's an ordered sequence of objects, a set with structure.
The sets {1, 2} and {2, 1} are equivalent, since all that matters for a set is its members.
In contrast, the ordered triple <1, 2, 5> is distinct from the ordered triple <2, 1, 5> which is distinct from the ordered triple <5, 2, 1> even though they all have the same members.

For the semantics of **F**, a two-place predicate is assigned sets of ordered pairs, a three-place predicate is assigned sets of three-place relations, etc.
Given a universe of {1, 2, 3}, the relation 'Gxy', which could be understood as meaning 'is greater than' would be standardly interpreted by {<2,1>, <3,1>, <3, 2>}

Our definitions of satisfaction and truth will need to be adjusted:
Objects in the domain can satisfy predicates; that remains the case for one-place predicates.
Ordered n-tuples may satisfy relational predicates.
A wff will be satisfiable if there are objects in the domain of quantification which stand in the relations indicated in the wff.
A wff will be true for an interpretation if all objects in the domain of quantification stand in the relations indicated in the wff.
And, still, a wff will be logically true if it is true for every interpretation.

For an example, let's extend the interpretation of a small set of sentences with a small domain that we considered when originally discussing semantics of **M**.

Sentences:
1. Pa • Pb
2. Wa • ~Wb
3. Oab
4. Obc
5. (∃x)(Px • Oxb)
6. (∃x)(Px • Obx)
7. (x)[Wx ⊃ (∃y)(Px • Oyx)]

Domain: {Bob Simon, Rick Werner, Katheryn Doran, Todd Franklin, Marianne Janack, Russell Marcus, Martin Shuster}

a: Katheryn Doran
b: Bob Simon
c: Russell Marcus

Px: {Bob Simon, Rick Werner, Katheryn Doran, Todd Franklin, Marianne Janack, Russell Marcus}
Wx: {Katheryn Doran, Marianne Janack}
Oxy: {<Bob Simon, Rick Werner>, <Bob Simon, Katheryn Doran>, <Bob Simon, Todd Franklin>, <Bob Simon, Marianne Janack>, <Bob Simon, Russell Marcus>, <Rick Werner, Katheryn Doran>, <Rick Werner, Todd Franklin>, <Rick Werner, Marianne Janack>, <Rick Werner, Russell Marcus>, <Katheryn Doran, Todd Franklin>, <Katheryn Doran, Marianne Janack>, <Katheryn Doran, Russell Marcus>, <Todd Franklin, Marianne Janack>, <Todd Franklin, Russell Marcus>, <Marianne Janack, Russell Marcus>, <Bob Simon, Martin Shuster>, <Rick Werner, Martin Shuster>, <Katheryn Doran, Martin Shuster>, <Todd Franklin, Martin Shuster>, <Marianne Janack, Martin Shuster>, <Russell Marcus, Martin Shuster>}

On this interpretation, 1 and 2 remain true; 3 is false while 4 is true; 5 is false but 6 and 7 are true.

III. Exercises A. Translate each sentence into predicate logic, using relational predicates.

1. John loves Mary
2. Tokyo isn't smaller than New York.
3. Marco was introduced to Paco by Erika.
4. America took California from Mexico.

IV. Quantifiers with relational predicates

Consider again the original argument.
We can now translate the first two premises and the conlcusion.

Bob is taller than Charles: Tbc
Andrew is taller than Bob: Tab
Andrew is taller than Charles: Tac

But we need quantifiers for the relations in the third premise.

Let's start with some sentences with just one quantifier.
The following four sentences use 'Bxy' for 'x is bigger than y'.

Joe is bigger than some thing : $(\exists x)Bjx$
Something is bigger than Joe: $(\exists x)Bxj$
Joe is bigger than everything: $(x)Bjx$
Everything is bigger than Joe: $(x)Bxj$

Next, we can introduce overlapping quantifiers.
Consider: 'Everything loves something', using 'Lxy' for 'x loves y': $(x)(\exists y)Lxy$
Note the different quantifier letters: overlapping quantifiers must use different variables.
Also, the order of quantifiers matters
'$(\exists x)(y)Lxy$' means that something loves everything, which is different.
Returning to the original argument, we now have:

1. Tbc
2. Tab
3. $(x)(y)(z)[(Txy \bullet Tyz) \supset Txz]$ / Tac

After we finish translations, we will return to deriving the conclusion of this argument.
Here are some more examples:

1. Something taught Plato. (Txy: x taught y)
   $(\exists x)Txp$
2. Someone taught Plato. (Px: x is a person)
   $(\exists x)(Px \cdot Txp)$
3. Plato taught everyone.
   $(x)(Px \supset Tpx)$
4. Everyone knows something. (Kxy: x knows y)
   $(x)[Px \supset (\exists y)Kxy]$
5. Jen reads all books written by Asimov. (Bx: x is a book; Wxy: x writes y; Rxy: x reads y; j: Jen; a: Asimov)
   $(x)[(Bx \bullet Wax) \supset Rjx]$
6. Some people read all books written by Asimov.
   $(\exists x)\{Px \bullet (y)[(By \bullet Way) \supset Rxy]\}$
7. Some people read all books written by some one.
   $(\exists x)\{Px \bullet (\exists y)\{Py \bullet (z)[(Bz \bullet Wyz) \supset Rxz]\}\}$
8. Honest candidates are always defeated by dishonest candidates. (Hx, Cx, Dxy: x defeats y)
   $(x)\{(Cx \cdot Hx) \supset (\exists y)[(Cy \cdot {\sim}Hy) \cdot Dyx]\}$
9. No mouse is mightier than himself. (Mx, Mxy: x is mightier than y)
   $(x)(Mx \supset {\sim}Mxx)$
10. Everyone buys something from some store. (Px, Sx, Bxyz: x buys y from z)
   $(x)[Px \supset (\exists y)(\exists z)(Sz \cdot Bxyz)]$
11. No store has everyone for a customer.
   ${\sim}(\exists x)\{Sx \cdot (y)[Py \supset (\exists z)Byzx]\}$ or $(x)\{Sx \supset (\exists y)[Py \cdot (z){\sim}Byzx]\}$

V. Exercises B. Translate each of the following into predicate logic, using relational predicates.

1. Everyone is wiser than someone. (Wxy: x is wiser than y)
2. Someone is wiser than everyone.
3. Some financier is richer than everyone. (Fx, Rxy: x is richer than y)
4. No deity is weaker than some human. (Dx, Hx, Wxy: x is weaker than y)
5. There is a store from which everyone buys something. (Px, Sx, Bxyz: x buys y from z)

VI. Solutions

Answers to Exercises A:
1. Ljm
2. ~Stn
3. Ipme
4. Tcam

Anwers to Exercises B:
1. (x)[Px ⊃ (∃y)(Py · Wxy)]
2. (∃x)[Px · (y)(Py ⊃ Wxy)]
3. (∃x)[Fx · (y)(Py ⊃ Rxy)]
4. ~(∃x)[Dx · (∃y)(Hy · Wxy)]   or   (x)[Dx ⊃ (y)(Hy ⊃ ~Wxy)]
5. (∃x){Sx · (y)[Py ⊃ (∃z)Byzx]}