Philosophy 240
Symbolic Logic

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Class 32: November 10
Translation Using Relational Predicates

Limits of Monadic Predicates

Consider:

1. Bob is taller than Charles.
2. Andrew is taller than Bob.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Tb
2. Ya
3. ???

/ Ta

Relational (Polyadic) Predicates

- Dyadic:
  - Txy: x is taller than y
  - Kxy: x knows y
  - Bxy: x believes y
  - Dxy: x does y
- Triadic:
  - Gxyz: x gives y to z
  - Kxyz: x kisses y in z
  - Bxyz: x is between y and z
- We can construct four-place and higher-place predicates, too.

Choosing Your Predicates

- Andrés loves Beatriz
  - La
  - Lab
- Camila gave David the earring.
  - Gc
  - Gcde
- By using a relational predicate, we reveal more logical structure.
- The more logical structure we reveal, the more we can facilitate inferences.

Full First-Order Logic

- We are now using **F**, for Full First-Order Predicate Logic, rather than **M**.
- For the purposes of this course, the differences between **F** and **M** are minor.
- Beyond this course, the differences between **M** and **F** are significant; we have breached a barrier.
- **M** admits of a decision procedure: there is a way of deciding, for any given formula, whether it is a theorem or not.
- **F** is not decidable.
- There are formulas for which there are no effective methods for deciding whether they are theorems or not.

Syntax for **M** and **F**

Vocabulary for **M** and **F**

Capital letters A...Z used as one-place predicates
Lower case letters (**terms**)
a, b, c,...u are used as constants.
v, w, x, y, z are used as variables.
Five connectives: ~, •, ∨, ⊃ ≡
Quantifier: ∃
Punctuation: ( ), [ ], { }

Formation Rules for Wffs of **M**

1. A predicate (capital letter) followed by a constant or variable (lower-case letter) is a wff.
2. If α is a wff, so are
   (∃x)α, (∃y)α, (∃z)α, (∃w)α, (∃v)α
   (x)α, (y)α, (z)α, (w)α, (v)α
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
5. These are the only ways to make wffs.

Formation Rules for Wffs of **F**

1. **An n-place predicate followed by n terms is a wff.**
2. If α is a wff, so are
   (∃x)α, (∃y)α, (∃z)α, (∃w)α, (∃v)α
   (x)α, (y)α, (z)α, (w)α, (v)α
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
5. These are the only ways to make wffs.

Semantics for F

- Recall that there were four steps for providing a standard formal semantics for **M**
  - Step 1. Specify a set to serve as a domain of interpretation, or domain of quantification.
  - Step 2. Assign a member of the domain to each constant.
  - Step 3. Assign some set of objects in the domain to each predicate.
  - Step 4. Use the customary truth tables for the interpretation of the connectives.
- The introduction of relational predicates requires adjustment to Step 3.
- We assign sets of ordered n-tuples to each relational predicate.

N-Tuples

- An n-tuple is an n-place relation.
  - an ordered sequence of objects
  - a set with structure
- {1, 2} = {2, 1}
- <1, 2, 5> ≠ <2, 1, 5> ≠ <5, 2, 1>
- An n-place predicate is assigned sets of ordered n-tuples
  - doubles, triples, quadruples...
- Gxy
  - Domain = {1, 2, 3}
  - {<2,1>, <3,1>, <3, 2>}

Satisfaction and Truth

- Objects in the domain (still) can satisfy one-place predicates.
- Ordered n-tuples may satisfy relational predicates.
- A wff will be satisfiable if there are objects in the domain of quantification which stand in the relations indicated in the wff.
- A wff will be true for an interpretation if all objects in the domain of quantification stand in the relations indicated in the wff.
- And, still, a wff will be logically true if it is true for every interpretation.

A Sample Theory and Interpretation

1. Pa • Pb
2. Wa • ~Wb
3. Oab
4. Obc
5. (∃x)(Px • Oxb)
6. (∃x)(Px • Obx)
7. (x)[Wx ⊃ (∃y)(Px • Oyx)]

- Domain: {Bob Simon, Rick Werner, Katheryn Doran, Todd Franklin, Marianne Janack, Russell Marcus, Martin Shuster}
- Constants
  - a: Katheryn Doran
  - b: Bob Simon
  - c: Russell Marcus
- Predicates
  - Px: {Bob Simon, Rick Werner, Katheryn Doran, Todd Franklin, Marianne Janack, Russell Marcus, Martin Shuster}
  - Wx: {Katheryn Doran, Marianne Janack}
  - Oxy: {<Bob Simon, Rick Werner>, <Bob Simon, Katheryn Doran>, <Bob Simon, Todd Franklin>, <Bob Simon, Marianne Janack>, <Bob Simon, Russell Marcus>, <Rick Werner, Katheryn Doran>, <Rick Werner, Todd Franklin>, <Rick Werner, Marianne Janack>, <Rick Werner, Russell Marcus>, <Katheryn Doran, Todd Franklin>, <Katheryn Doran, Marianne Janack>, <Katheryn Doran, Russell Marcus>, <Todd Franklin, Marianne Janack>, <Todd Franklin, Russell Marcus>, <Marianne Janack, Russell Marcus>, <Bob Simon, Martin Shuster>, <Rick Werner, Martin Shuster>, <Katheryn Doran, Martin Shuster>, <Todd Franklin, Martin Shuster>, <Marianne Janack, Martin Shuster>, <Russell Marcus, Martin Shuster>}
- 1 and 2 are true.
- 3 is false while 4 is true.
- 5 is false but 6 and 7 are true.

Some Translations

1. John loves Mary
2. Tokyo isn't smaller than New York.
3. Marco was introduced to Paco by Erika
4. America took California from Mexico.

Our Original Argument

Consider:

1. Bob is taller than Charles.
2. Andrew is taller than Bob.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Tbc
2. Tab
3. ???

/ Tac

Quantifiers and Relational Predicates

Bxy: x is bigger than y

- Joe is bigger than some thing.
  $(\exists x)Bjx$
- Something is bigger than Joe.
  $(\exists x)Bxj$
- Joe is bigger than everything.
  $(x)Bjx$
- Everything is bigger than Joe.
  $(x)Bxj$

Overlapping Quantifiers

Lxy: x loves y

- Everything loves something.
  (x)(∃y)Lxy
- Something loves everything.
  (∃x)(y)Lxy
- (x)(∃y)Lyx
- (∃x)(y)Lyx

Our Original Argument

Finally Translated

Consider:

1. Bob is taller than Charles.
2. Andrew is taller than Bob.
3. For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.

So, Andrew is taller than Charles.

1. Tbc
2. Tab
3. (x)(y)(z)[(Txy • Tyz) ⊃ Txz] / Tac

More Examples

1. Something taught Plato. (Txy: x taught y)
2. Someone taught Plato. (Px: x is a person)
3. Plato taught everyone.
4. Everyone knows something. (Kxy: x knows y)
5. Jen reads all books written by Asimov. (Bx: x is a book; Wxy: x writes y; Rxy: x reads y; j: Jen; a: Asimov)
6. Some people read all books written by Asimov.
7. Some people read all books written by some one.
8. Honest candidates are always defeated by dishonest candidates. (Hx, Cx, Dxy: x defeats y)
9. No mouse is mightier than himself. (Mx, Mxy: x is mightier than y)
10. Everyone buys something from some store. (Px, Sx, Bxyz: x buys y from z)
11. No store has everyone for a customer.

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