Philosophy 240: Symbolic Logic Fall 2010 Mondays, Wednesdays, Fridays: 9am - 9:50am Hamilton College Russell Marcus rmarcus1@hamilton.edu

Class 27 - October 29 Philosophy Friday #5: Truth and Liars Tarski, "The Semantic Conception of Truth and the Foundations of Semantics"

Given that philosophers often think of their discipline as the search for truth, it may come as no surprise that we spend a lot of time thinking about the nature of truth itself.

Unfortunately, philosophical discussions of truth can quickly become difficult and obscure.

There is a lot of technical work on truth centering around responses to semantic paradoxes.

There are also more fundamental questions about the nature of truth and our ability to know what is true. Technical work on the logic of truth is sophisticated.

Less-technical discussions of truth often rely on interpretations of the technical results.

We will start with a general overview of three non-technical theories of truth, asking whether truth is a property, and, if so, what kind of property.

Then, we will look at the semantic paradoxes, and ask why they are important.

Lastly, we examine Tarski's important work on truth, and his solution to the problems raised by the paradoxes.

I. Truth

The standard concept of truth is called the correspondence theory.

The correspondence theory of truth traces back at least to Plato, though it is traditional to ascribe it to Aristotle.

To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true (*Metaphysics*, 1011b25).

According to the correspondence theory of truth, truth is a relation between words and worlds. The truth of a sentence consists in its agreement with, or correspondence to, reality.

One worry about the correspondence theory is that we do not seem to have any extra-linguistic way to apprehend reality.

If I want to compare, say, an elephant to a picture of an elephant, or a picture of a sculpture of an elephant to a picture of an elephant, I can hold both of them in front of me, gazing from the one to the other.





If I want to compare my words to the world, I have to apprehend, on the one side, what the words mean, and on the other, the world.

But, it has seemed to some philosophers, I only apprehend the world mediately, through my ideas of it. I do not have any access to the world as it is in itself.

(Those of you who have worked through the epistemology of the modern era will have a good understanding of the problem here.)

It seems as if I am unable to compare my words, or my ideas, to an independent world, to decide whether there really is a correspondence between them.

The correspondence theory says that truth is a matching of words to the world, but I can only really know about one side of the equation.

In response to such problems with the correspondence theory of truth, some philosophers have adopted coherence theories.

According to coherentism, the truth of a sentence consists in its consistency with other beliefs we hold. Different people apprehend the world in different ways, depending on their experiences, expectations, and background beliefs.

The coherentist despairs of any method of resolving these inconsistencies among people and their beliefs.

Imagine that I believe in a traditional, monotheistic God and that you do not. 1 will be true for me, since it coheres with my other beliefs.

1. God is omniscient.

In contrast, 1 will be false for you, since it conflicts with your other beliefs.

Since different people hold different beliefs, the coherence-truth of a sentence depends on the person who is considering the sentence.

Coherence theories thus collapse into relativism.

The correspondence and coherence theories of truth each provide a univocal analysis of 'truth'. Insofar as they entail that there is a property called truth, they are both inflationary theories of truth. Inflationary theories are distinguished from deflationary theories of truth.

Deflationary theories of truth were developed in the last century, and are often called minimalist theories. Deflationary theories have many proponents, all of whom have different ways of understanding and explaining deflationism.

Deflationists are united in the belief that there is no essence to truth, no single reduction of truth to a specific property, like correspondence or consistency.

Some deflationists claim that truth is just a device for simplifying long conjunctions. If you said a lot of smart things at the party, I could list them all. Or, I could just say:

2. Everything you said last night was true.

In 2, 'true' is eliminable by a long set of sentences listing all of what you said last night.

Such eliminations are, according to the deflationist, the central purpose of 'truth'.

Otherwise, 'truth' is merely a redundant term.

Indeed, deflationism is often called a redundancy theory of truth: to say that snow is white is true is just to say, redundantly, that snow is white.

Both inflationists and deflationists agree that a minimal condition for truth is what we call the T-schema, or Convention T, following Tarski.

3. p is true iff x

In 3, 'p' is the name of any sentence, and x are the truth conditions of that sentence.

We can use the T-schema to specify the truth conditions for any sentence. Here are some instances of the T-schema:

- 4. 'The cat is on the mat' is true iff the cat is on the mat.
- 5. '2+2=4' is true iff 2+2=4
- 6. 'Barack Obama is president' is true iff the husband of Michelle Obama and father of Sasha Obama and Malia Obama is head of the executive branch of the United States of America.

Note that, as in 6, the truth conditions need not be expressed in the same terms as the sentence on the left. We can even use a different language for the sentence and for its truth conditions. Consider:

7. 'El gato está en el alfombrilla' is true iff the cat is on the mat.

Notice that you could understand the truth conditions of 7 without understanding the meaning of the Spanish sentence on the left side.

Inflationists and deflationists disagree about whether the T-schema is all there is to know about truth. The inflationist believes that there are explanations of the concept of truth inherent in the truth conditions on the right side of the T-schema.

For the correspondence theorist, 'the cat is on the mat' is true because there is a cat, which corresponds to 'the cat', and there is a mat, which corresponds to 'the mat', and there is a relation, being on, which the cat and the mat satisfy, or in which they stand.

All other instances of the T-schema will have similar explanations in terms of the correspondence of words to worlds.

The deflationist, in contrast, believes that the T-schema is all there is to know about truth, and that there is no single kind of explanation of why all sentences are true.

'Truth' varies in application.

Again, according to the deflationist, we do not even need 'true' in our language. It's just a handy tool.

Tarski, one of the great twentieth-century logicians, introduced the T-schema as an essential component of his treatment of the semantic paradoxes.

There is some debate about whether Tarski is best understood as a deflationist or as an inflationist, to which we will return after looking in more detail at how he dealt with the paradoxes.

II. The Liar, and Other Semantic Paradoxes

In Philosophy Friday #4: Three-Valued Logics, we discussed eight motivations for three-valued logics. One important motivation for introducing a third truth-value was to deal with the semantic paradoxes. The most important semantic paradox is called the liar.

8. This sentence is false.

8 is an example of a paradoxical sentence.If 8 is true, then it is false, which makes it true, which makes it false...8 lacks a single, definite truth value, even though it is a grammatically well-formed sentence.

The liar is often called Epimenides' paradox. Epimenides was a Cretan to whom the statement that all Cretans are liars is attributed.

Quine, in his essay "The Ways of Paradox," argues that there are grounds to question either the paradoxicality or the well-formedness of 8. It is not clear what 'this sentence' refers to. If we substitute 'this sentence is false' for 'this sentence', then we get

9. 'This sentence is false' is false.

9 does not ascribe falsity to itself, and the paradox is gone, or at least delayed. But, we can find other, similarly troublesome sentences. Quine constructed 10, which avoids the above problem.

10. 'Yields falsehood when appended to its own quotation' yields falsehood when appended to its own quotation.

In both 8 and 10, the culprit seems to be falsity.

Truth and falsity are called semantic terms.

'Semantic' can refer to truth or to meaning.

In ordinary usage, when we talk about semantics, we refer to meanings.

When we present a semantics for a formal language, we provide truth conditions for the wffs of the language.

Our semantics for PL, for example, consisted of truth tables.

Soon we will look at the more complicated semantics for M.

The problem with many sentences like 8 and 10 seems to be rooted in the presence of semantic terms, like 'true' and 'false'.

Thus, such problematic sentences are called semantic paradoxes.

One diagnosis of many semantic paradoxes, including the liar, is that they involve illicit self-reference. Another self-referential paradox, the barber paradox, is due to Bertrand Russell, though he credits an anonymous source.

Consider the barber in a town who shaves all the men who do not shave themselves. Does he shave himself?

You can construct a puzzling declarative sentence, similar to the liar, which I leave to you as an exercise.

But, not all semantic paradoxes involve truth, or self-reference. Consider Grelling's paradox:

Some predicates apply to themselves, whereas others do not.

'Polysyllabic' is polysyllabic; 'monosyllabic' is not monosyllabic.

Call a predicate heterological if it does not apply to itself.

'Monosyllabic' is heterological; 'polysyllabic' is not heterological.

(We can call it autological, or homological.)

Now, consider whether 'heterological' applies to itself.

If it does, then 'heterological' is not heterological.

But, if 'heterological' is not heterological, then it does not apply to itself, which means that it is heterological.

We can construct a statement involving 'heterological' whose truth value is puzzling.

11. 'Heterological' is heterological.

Grelling's paradox is semantic, but does not involve 'truth' or 'falsity' explicitly. Grelling's paradox is about meaning.

There are two popular solutions to the semantic paradoxes:

S1. Introduce a third truth value for paradoxical sentences.

S2. Banish semantic terms from formal languages.

There are two problems with S1.

First, as we saw, systems of three-valued logic either lose logical truths and valid inferences (on B and K_3) or ascribe truth to conditional sentences with indeterminate antecedents and consequences (on L_3). Second, adding a third truth value will not solve the problem of the strengthened liar.

12. This sentence is not true.

If 12 is true, then since it says that it is not true, it must be either false or indeterminate. But, if it is false or indeterminate, then what 12 says holds of itself. So, 12 is true. The paradox recurs.

I'll put aside S1, since we have already looked at three-valued logics. The second popular solution, S2, is Tarski's, which we will examine in a moment. First, we should look in greater detail how the paradoxes create logical difficulties.

III. Explosion, or What's So Bad About the Paradoxes?

In the early twentieth century, truth had gotten a terrible reputation, in large part due to the paradoxes. The paradoxes lead to contradictions.

Contradictions are unacceptable in traditional, or classical, formal systems because a contradiction entails anything.

This property of classical systems is called explosion.

Explosion	1. P • ~P	/ Q
	2. P	1, Simp
	3. $\mathbf{P} \lor \mathbf{Q}$	2, Add
	4. ~P	1, Com, Simp
	5. Q	3, 4, DS
QED	-	

To see how the liar leads to a contradiction, consider 8 again. Applying the T-schema yields:

13. 8 is true iff 8 is false.

We can translate this sentence into \mathbf{M} by taking a constant, say 'p', to stand for the sentence 8, and introducing a truth predicate, 'T'.

We also have to take 'P is true' to be the negation of 'P is false'; the strengthened liar will work a bit differently.

We get:

14.	1. Tp \equiv ~Tp	From the T-schema and the definition of P	
	2. $(Tp \supset \neg Tp) \bullet (\neg Tp \supset Tp)$	1, Equiv	
	3. $\sim Tp \supset Tp$	2, Com, Simp	
	4. Tp \lor Tp	3, Impl, DN	
	5. Tp	4, Taut	
	6. Tp $\supset \sim$ Tp	2, Simp	
	7. $\neg Tp \lor \neg Tp$	6, Impl	
	8. ~Tp	7, Taut	
	9. Tp • ~Tp	5, 8, Conj	
Tilt!		-	

Our natural language contains the word 'true', as a predicate.

If we include a truth predicate in our formal language, we can construct the liar sentence.

If we can construct the liar sentence, we can formulate an explicit contradiction.

Contradictions explode.

Everything is derivable.

But, we know that not every sentence is true.

So, if we include a truth predicate in our formal language, our formal language will not be able to contain, or reveal, our true commitments.

The excitement surrounding the new logic of the early twentieth century included hopes that all human knowledge could be represented by formal languages, like the logic we are studying.

Since contradictions lead to explosion, and formal languages in which the paradoxes are representable lead to contradictions, it became seen as essential to avoid formalizing the notion of truth as I did in 14. Since formal languages were seen as the locus of all of our knowledge, it seemed that truth was just not a legitimate term, not something that we could know.

The bad reputation of truth explains, at least in part, the interest of many philosophers in the relativism of coherence truth.

All recent work on truth, whether deflationary or inflationary, owes its origins to Tarski, who, in the 1930s, showed how to rehabilitate the concept of truth within formalized languages, how to avoid explosion without giving up on a formalized notion of truth.

IV. Tarski's Solution

Tarski's solution to the liar paradox is to distinguish between an object language and a meta-language, and to rule sentence 8 out of the object language.

The object language is the language in which we are working.

The meta-language is a language in which we can talk about the object language.

Instances of the T-schema are sentences of the meta-language which we can use to characterize truth for the object language.

Since we are constructing a formal language, like PL or M, we can include or exclude any terms.

Deleting just the liar from the object language might appear arbitrary and ad hoc.

Tarski claims that the paradoxes show that all uses of the term 'true', and related semantic terms, are illegitimate within any object language.

We can not, on pain of contradiction, construct the truth predicate for a language within that language. As long as we ascend to a meta-language, we can construct a truth predicate for any object language. To determine which sentences of an object language are true and which are false, we have to examine the

truth conditions as given on the right hand side of instances of the T-schema.

While the sentences themselves are elements of the object language, the truth conditions are, technically, written in the meta-language.

The key to Tarski's solution to the liar paradox is that sentences like 8 are ill-formed, since they include 'false' in the object language.

When I want to use a sentence like 2, he claims, I implicitly ascend to a meta-language to do so. In a meta-language, I can also construct sentences like the important 15.

15. All consequences of true sentences are true.

Sentences like 15 are fundamental to metalogic, and model theory, fields that Tarski more or less created. Metalogic is an advanced logic topic, though we saw some in our discussion of adequacy.

In metalogic, we explore questions of whether a formal system is sound, or complete, or decidable. We will put them aside, here, and see if Tarski's T-schema can help us understand our ordinary conception of truth.

Before we approach the questions about whether Tarski's theory is inflationary or deflationary, about how we are to understand the concept of 'truth' more broadly, we should look briefly at a technical concern about the sufficiency of his solution, and an alternative.

V. A Kripkean Alternative

Tarski's construction produces a hierarchy of languages.

To construct a truth predicate for an object language, we eliminate semantic terms from our object language and ascend to a meta-language.

For our purposes in this class, that means that we will not include a truth predicate as part of \mathbf{M} . If we want to know about the truth of sentences of \mathbf{M} , we take an external perspective, working in a meta-language.

We might reasonably wonder about truth in the meta-language.

Of course, for the same reasons that the object language can not contain a truth predicate, the metalanguage can not contain its own truth predicate.

But, we can construct a truth predicate for the meta-language in a further meta-language. To construct a truth predicate for the second meta-language, we can construct a third, and so on. Each of the separate truth predicates occurs at a different level in this ever-expanding hierarchy.

The relations among these truth predicates are merely analogical.

Each meta-language is distinct, and has different terms.

Each truth predicate is independent of each other.

We are burdened not with one 'truth', but an infinite hierarchy of 'truth's.

To make matters worse, there are cases in which we do not know which level in the hierarchy any particular use of 'true' or 'false' belongs to. Consider:

16. Everything George W. Bush said was false.

16 must be made in a meta-language one step higher than anything that Bush ever said. So, to know what level my 'false' belongs to, I need to know about all the levels of Bush's uses of 'true' and 'false'.

If Bush once claimed:

17. Everything Bill Clinton said was false.

then in order to know what level Bush's 'false' occurs at, we also need to know all the levels of the uses of 'true' and 'false' in whatever Clinton said.

Furthermore, if Clinton were the speaker of 16, then Bush and Clinton become embroiled in an ugly semantic circle.

Kripke, in a paper called "Outline of a Theory of Truth," showed that we can construct a truth predicate for a language embedded within the object language itself, without creating paradox. Here is a far-too-quick sketch of Kripke's approach:

We start with a base language, containing no logical connectives, quantifiers, or truth predicate. Then, we add a truth predicate to the language itself.

We can more-or-less easily decide which sentences of the base language are true and which are false, since there is no truth predicate in the base language.

Then, we can add the familiar logical connectives: negation, conjunction, and disjunction, say. The semantics for the propositional connectives are easily presented, as well.

So, we can apply the truth predicate to all base-level sentences and logical functions of them. Next, we can consider sentences with single uses of semantic terms licensed so far.

We repeat the original process, adding more complex sentences to our lists of true and false sentences. We can proceed to sentences of greater and greater semantic complexity.

At each level, we bring along all the earlier sentences, and apply the truth predicate to them.

But, the truth predicate does not apply to sentences at its own level.

Eventually, we can, in principle, reach any of a variety of fixed points past which further construction is unwarranted.

There are many different fixed points, and lots of technical work can be done with them.

The key to Kripke's construction is that he produces an object-level truth predicate.

This object-language truth predicate allows us to value many sentences that include 'true'.

We can have a language with all of those sentences, and one truth predicate for all of them.

Kripke contains the entire Tarskian hierarchy in one language.

Kripke's paper is technical, but extremely elegant.

Those with mathematics or other technical backgrounds might enjoy working on a paper about it. For now, I will put aside Kripke's improvements on Tarski's technical work, and the problems of the hierarchy of truth predicates, and return to the philosophical questions about truth raised by Tarski. Most importantly, how does the technical work on truth illuminate the deeper issues about the nature of truth?

VI. Is Truth Deflationary or Inflationary?

Tarski calls the notion of truth which underlies his method of introducing a truth predicate into a metalanguage the semantic conception of truth.

He uses sentences like 2 and 15 to show that 'truth' plays an essential role in a theory.

It might thus seem like Tarski is an inflationist, a correspondence theorist.

But Tarski's claim that 'truth' is essential may not have inflationary implications.

If 'true' is a device used to refer to other sentences, it depends on what we think of those other sentences, the ones without 'true' and with content.

If we need a words-worlds relation in order to ascribe 'true' to a sentence, then truth will not be merely deflationary, or redundant.

If all there is to truth is eliminable, then perhaps there is no essence to truth.

Aristotle's original claim could itself be given a deflationary interpretation!

Tarski prescribes the following method to determining the correct notion of truth.

It seems to me obvious that the only rational approach to [questions about the correct notion of truth] would be the following: We should reconcile ourselves with the fact that we are confronted, not with one concept, but with several different concepts which are denoted by one word; we should try to make these concepts as clear as possible (by means of definition, or of an axiomatic procedure, or in some other way); to avoid further confusions, we should agree to use different terms for different concepts; and then we may proceed to a quiet and systematic study of all concepts involved, which will exhibit their main properties and mutual relations (355).

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Furthermore, Tarski believes that the semantic conception is agnostic among any deeper philosophical debates.

We may accept the semantic conception of truth without giving up any epistemological attitude we may have had; we may remain naive realists, critical realists or idealists, empiricists or metaphysicians - whatever we were before. The semantic conception is completely neutral toward all these issues (362).

Deflationists look at the T-schema as a satisfactory definition of truth.

That's why deflationism also goes by the name 'redundancy theory'.

Inflationists about truth look at the T-schema as merely a minimal condition for truth.

They claim that there are additional requirements, like correspondence to reality.

Hartry Field's paper "Tarski's Theory of Truth" argues convincingly that Tarski is not a deflationist. It is also a difficult, technical paper, but influential and fecund, and more strictly philosophical than Kripke's paper.

Field shows that in order to use the T-schema as a definition of truth, we need to supplement it with some kind of account of why we choose certain sentences to be true and not others.

To see the problem, note that we could understand the truth conditions in 7 without understanding the Spanish sentence on the left.

To capture truth, it is not enough just to list the true and false sentences of a language.

We want to analyze the component parts of the Spanish expressions, and how they interact to form true or false sentences.

The T-schema, by itself, does not provide that kind of explanation.

Tarski's construction only reduces 'truth' to other semantic notions.

If we are merely concerned with constructing a meta-linguistic truth predicate, the T-schema might suffice.

We might, in contrast, wish to take Tarski's claim to constructing a semantic notion of truth seriously. In that case, we need not merely to explain truth in terms of other semantic notions, but to show how sentences become either true or false.

We would like, in addition to the T-schema, an explanation of why the terms are true of the things of which they are true, in a way that is consistent with our other scientific commitments.

It is not that we could not add such an account to complete Tarski's theory.

But, once we do, the theory does not appear deflationary.

We started by wondering about the nature of truth, whether it is correspondence to reality, or consistency, or whether it lacks any univocal nature.

The question has now become whether Tarski's formalized semantic conception captures our ordinary notion.

Is there more to be said about truth than Convention T?

VII. Did Tarski Present the Final Word on Truth?

There are at least two ways to look at Tarski's semantic theory of truth.

The first way is minimalist, and it focuses on the condition of adequacy, the T-schemas.

The second way is inflationist, and it focuses on the extent to which Tarski legitimizes our ordinary, correspondence notion of truth.

There is no question that the notion of truth is useful, in sentences like 2, and essential to metalogical work, in sentences like 15.

Tarski, and those following him, have vindicated formal theories of truth insofar as they allow us to capture these minimal uses of the term.

The question of whether philosophers need an inflationary notion of truth continues to be discussed. Tarski makes a compelling argument that science aims at truth.

The main reason we want consistent theories is because we know that an inconsistent theory contains a falsehood.

As soon as we succeed in showing that an empirical theory contains (or implies) false sentences, it cannot be any longer considered acceptable (102).

Some philosophers try to replace truth with warranted assertability, or coherence, or some other conditions.

Personally, I'm a big fan of truth.

But, there are obvious epistemic worries about our access to it.

The old problem of whether we can assess a words-worlds connection, being embedded in one side of it, still resonates.

As I mentioned, much of the contemporary work on truth and the paradoxes is technical, though the classical discussion of theories of truth is mainly philosophical.

Michael Lynch's *True to Life: Why Truth Matters* is a friendly introduction to the non-technical work on truth by someone who has worked with the contemporary questions.

One of the more controversial but productive areas of recent research has been dialetheism.

According to dialetheists like Graham Priest, the liar is both true and false.

There has been a lot of technical work on paraconsistent logics, logics which contain contradictions.

Contradictions in classical logic are explosive: anything follows.

So, dialetheists look to block explosion.

A paper on dialetheism could be interesting.

Whether or not Tarski's solution to the problem of the paradoxes is ideal, the distinction between object language and meta-language has become fundamental in all contemporary treatments of logic. I have carefully presented precise rules for the formation of our object language, which is our proper domain of study.

The Greek letters I use to describe argument forms, and the truth values \top and \bot , are all elements of the meta-language we use to study the object language.

The formalization of this distinction traces directly to Tarski's work on truth.

VIII. Paper Topics

1. Does introducing a third truth-value solve the problem of the liar? Discuss the strengthened liar paradox. Kirkham has a good, if brief, discussion of the strengthened liar.

2. Is truth deflationary or inflationary? See Horwich, and the Lynch collection. Fisher has a fine introductory discussion.

3. Is truth relative to a language? Tarski's definition of truth introduces a different truth predicate for each language, and creates a hierarchy of languages. Is this construction objectionable? See Fisher, Kirkham, and the Lynch collection.

4. Graham Priest has lately been defending dialetheism, the claim that there can be true contradictions. Can there be true contradictions? Is the liar one of them?

5. For a more technical paper, describe the difference between Kripke's truth predicate and Tarski's hierarchy. What advantages does Kripke claim for his construction? Is it satisfactory?

6. Is truth a correspondence between words and reality? See the Lynch collection for the classic, historical discussion.

IX. Suggested Readings

- Aristotle. *Metaphysics*. In *The Complete Works of Aristotle, vol.2*, Jonathan Barnes, ed. Princeton University Press, 1984. The definition of truth is at 1011b25.
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Haack, Susan. Philosophy of Logics. Chapters 7 and 8.

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- Quine, Willard van Orman. "The Ways of Paradox." Harvard University Press, 1976. The title essay is the source of the 'yields a falsehood...' paradox, and contains an excellent discussion of paradoxes.
- Tarski, Alfred. "The Semantic Conception of Truth and the Foundations of Semantics." *Philosophy and Phenomenological Research* 4.3: March 1944, pp 341-376.