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More Derivations in Predicate Logic and Changing Quantifiers (§8.2 - §8.3)

I. More complex derivations

You may not instantiate a line on which the quantifier is not the main operator.
In this example, line 2 can not be instantiated.

1. (x)(Dx · Ex)		
2. (x)Dx ⊃ Fa		/ (∃x)Fx
3. Dx · Ex	1, UI	
4. Dx	3, Simp	
5. (x)Dx	4, UG	
6. Fa	2, 5, MP	
7. (∃x)Fx	6, EG	

QED

Similarly, we we can not take off either quantifier in line 1 of the following derivation.

1. (x)(Jx ∨ Kx) ⊃ (∃y)Ly	
2. (x)(Jx ∨ Lx)	
3. (x)(~Lx ∨ Kx)	/ (∃x)Lx
4. Jx ∨ Lx	2, UI
5. ~Jx ⊃ Lx	4, DN, Impl
6. ~Lx ∨ Kx	3, UI
7. Lx ⊃ Kx	6, Impl
8. ~Jx ⊃ Kx	5, 7, HS
9. Jx ∨ Kx	8, Impl, DN
10. (x)(Jx ∨ Kx)	9, UG
11. (∃x)Lx	1, 10, MP

QED

You may instantiate the same quantifier twice.
When that quantifier is universal, there are no restrictions.

1. (x)(Mx ⊃ Nx)	
2. (x)(Nx ⊃ Ox)	
3. Ma · Mb	/ Na · Ob
4. Ma ⊃ Na	1, UI
5. Ma	3, Simp
6. Na	4, 5, MP
7. Mb ⊃ Nb	1, UI
8. Mb	3, Com, Simp
9. Nb	7, 8, MP
10. Nb ⊃ Ob	2, UI
11. Ob	10, 9, MP
12. Na · Ob	6, 11, Conj

QED

When the quantifier is existential, the second instantiation must go to a new constant.

1. $(\exists x)(Px \bullet Qx)$	
2. $(x)(Px \supset Rx)$	/ $Ra \bullet Qb$
3. $Pa \bullet Qa$	1, EI
4. $Pa \supset Ra$	2, UI
5. Pa	3, Simp
6. Ra	4, 5, MP
7. $Pb \bullet Qb$	1, EI
8. Qb	7, Com, Simp
9. $Ra \bullet Qb$	6, 8, Conj

QED

II. Exercises A. Derive the conclusions of each of the following arguments.

1.
1. $(x)(Mx \supset Nx)$
2. $\sim Na$ / $\sim Ma$

2.
1. $(x)(Ox \supset \sim Px)$
2. $(\exists x)(Rx \cdot Px)$ / $(\exists x)(Rx \cdot \sim Ox)$

3.
1. $(\exists x)Sx \supset (x)Tx$
2. $(\exists x)Ux \supset (\exists x)Wx$
3. $Sb \cdot Ub$ / $(\exists x)(Wx \cdot Tx)$

4.
1. $(\exists x)Gx \supset (y)(\sim Hy \vee Iy)$
2. Gc
3. $\sim If$ / $(\exists x)\sim Hx$

5.
1. $(\exists x)Ax \supset (x)(Bx \supset Ex)$
2. $(\exists x)Dx \supset (\exists x)\sim Ex$
3. $(\exists x)(Ax \cdot Dx)$ / $(\exists x)\sim Bx$

III. Exercises B. Match each sentence in the left column with its equivalent in the right column.

1. Everything is made of atoms.	A. Not everything is made of atoms.
2. Something is made of atoms.	B. It's wrong to say that nothing is made of atoms.
3. Nothing is made of atoms.	C. It's not the case that something is not made of atoms.
4. At least one thing isn't made of atoms.	D. It's false that something is made of atoms.

Solutions: 1C, 2B, 3D, 4A

IV. Changing quantifiers

Look at the predicate logic regimentations of each set of equivalent pairs.

$(x)Ax$	is equivalent to	$\sim(\exists x)\sim Ax$
$(\exists x)Ax$	is equivalent to	$\sim(x)\sim Ax$
$(x)\sim Ax$	is equivalent to	$\sim(\exists x)Ax$
$(\exists x)\sim Ax$	is equivalent to	$\sim(x)Ax$

The rule of **Changing Quantifiers (CQ)**: Any place where you have an expression of one of the above forms, you may replace it with a statement of its logically equivalent form.

Like rules of replacement, CQ is based on logical equivalence, rather than validity, and thus may be used on part of a line.

Another way to look at these four rules.
There are three spaces around each quantifier:

1. Directly before the quantifier
2. The quantifier itself
3. Directly following the quantifier

CQ says that to change a quantifier, you change each of the three spaces.

Add or remove a negation directly before the quantifier.
Switch quantifiers: existential to universal or vice versa.
Add or remove a negation directly after the quantifier.

V. Some transformations permitted by CQ

'It's not the case that every P is Q' is equivalent to 'something is P and not Q'.

$\sim(x)(Px \supset Qx)$	
$(\exists x)\sim(Px \supset Qx)$	CQ
$(\exists x)\sim(\sim Px \vee Qx)$	Impl
$(\exists x)(Px \cdot \sim Qx)$	Dm, DN

'It's not the case that something is both P and Q' is equivalent to 'everything that's P is not Q,' and to 'everything that's Q is not P'.

$\sim(\exists x)(Px \cdot Qx)$	
$(x)\sim(Px \cdot Qx)$	CQ
$(x)(\sim Px \vee \sim Qx)$	DM
$(x)(Px \supset \sim Qx)$	Impl
$(x)(Qx \supset \sim Px)$	Trans, DN

VI. Sample derivations using CQ

1.	1. (∃x)Lx ⊃ (∃y)My		
	2. (y)~My	/~La	
	3. ~(∃y)My	2, CQ	
	4. ~(∃x)Lx	1, 3, MT	
	5. (x)~Lx	4, CQ	
	6. ~La	5, UI	Note: You may not use EI to get this conclusion!
QED			

2.	1. (x)[(Ax · Bx) ⊃ Ex]		
	2. ~(x)(Ax ⊃ Ex)	/ ~(x)Bx	
	3. (∃x)~(Ax ⊃ Ex)		2, CQ
	4. (∃x)~(~Ax ∨ Ex)		3, Impl
	5. (∃x)(Ax · ~Ex)		4, DM, DN
	6. Aa · ~Ea		5, EI
	7. (Aa · Ba) ⊃ Ea		1, UI
	8. ~Ea		6, Com, Simp
	9. ~(Aa · Ba)		7, 8, MT
	10. ~Aa ∨ ~Ba		9, DM
	11. Aa		6, Simp
	12. ~Ba		10, 11, DN, DS
	13. (∃x)~Ba		12, EG
	14. ~(x)Bx		13, CQ
QED			

3.	1. (x)~Dx ⊃ (x)Ex		
	2. (∃x)~Ex /(∃x)Dx		
	3. ~(x)Ex	2, CQ	
	4. ~(x)~Dx	1, 3, MT	
	5. (∃x)Dx	4, CQ	
QED			Note: No instantiation!

Exercises B. Derive the conclusions of each of the following arguments.

1.	1. ~(∃x)Hx	
	2. (x)~Hx ⊃ (z)Iz	/ Ia
2.	1. (∃x)(Hx · Gx) ⊃ (x)Ix	
	2. ~Ia	/ (x)(Hx ⊃ ~Gx)
3.	1. (∃x)(Ax ∨ Bx) ⊃ (x)Dx	
	2. (∃x)~Dx	/ ~(∃x)Ax
4.	1. (x)~Fx ⊃ (x)~Gx	/ (∃x)Gx ⊃ (∃x)Fx

5.
1. $(\exists x)\sim Ax \supset (x)\sim Bx$
2. $(\exists x)\sim Ax \supset (\exists x)Bx$
3. $(x)(Ax \supset Fx)$ / $(x)Fx$

Solutions may vary.