Philosophy 240
Symbolic Logic

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Translation into Predicate Logic II (§8.1)

Monadic and Relational Predicate Logics

- Predicate logic is monadic if the predicates only take one object.
- When predicates take more than one object, we call the predicates relational.
- Andrés loves Beatriz
  - Monadic: La
  - Relational: Lab
    - ‘Lxy’: x loves y:
- Relational predicates will allow us greater generality.
- We will look to reveal as much logical structure as we can.

Extensions of Monadic Predicate Logic

- Full First-Order Predicate Logic
- A specific predicate for identity
- Functors
- Second-order quantifiers (predicate variables)

Names of Languages We Will Study

- **PL**: Propositional Logic
- **M**: Monadic (First-Order) Predicate Logic
- **F**: Full (First-Order) Predicate Logic
- **FF**: Full (First-Order) Predicate Logic with functors
- **S**: Second-Order Predicate Logic

Languages and Systems of Deduction

- With PL, we used one language, and one set of inference rules.
- But we can use the same language in different deductive systems, and we can use the same deductive system with different languages.
- We will use **M** and **F** with the same deductive system.
- Then, we will introduce a new deductive system using the language **F**, adding new rules covering a special identity predicate.

Vocabulary of M

- Capital letters A...Z used as one-place predicates
- Lower case letters
  - a, b, c,...u are used as constants.
  - v, w, x, y, z are used as variables.
- Five connectives: ~, •, ∨, ⊃ ≡
- Quantifier: ∃
- Punctuation: ( ), [ ], { }
- Constants and variables are called **terms**.

On Scope

- Compare:
  1. (x)(Px ⊃ Qx) Every P is Q
  2. (x)Px ⊃ Qx   If everything is P, then x is Q
- The difference between these two expressions is the scope of the quantifier.

Scope of a Negation

(in propositional logic)

Whatever directly follows the negation symbol is in its scope.

- If what follows the negation is a single propositional variable, then the scope of the negation is just that propositional variable.
- If what follows the negation is another negation symbol, then the scope of the first negation symbol is the scope of the second negation symbol plus that second negation symbol.
- If what follows the negation is a bracket, then the entire formula which occurs between the opening and closing of that bracket is in the scope of the negation.

Scope of a Quantifier

Whatever formula immediately follows the quantifier is in its scope.

- If what follows the quantifier is a bracket, then any formulas that occur until that bracket is closed are in the scope of the quantifier.
- If what follows the quantifier is a negation, then every formula in the scope of the negation is in the scope of the quantifier.

Binding

- Quantifiers bind every instance of their variable in their scope.
- A **bound variable** is attached to the quantifier which binds it.
  1. (x)(Px ⊃ Qx)
  2. (x)Px ⊃ Qx
  - In 1, the ‘x’ in ‘Qx’ is bound.
  - In 2, the ‘x’ in ‘Qx’ is not bound.
- An unbound variable is called a **free variable**.
  3. (x)Px ∨ Qx
  4. (∃x)(Px ∨ Qy)
  - In 3, ‘Qx’ is not in the scope of the quantifier, so is unbound.
  - In 4, ‘Qy’ is in the scope of the quantifier, but ‘y’ is not the quantifier variable, so is unbound.

Formation Rules for Wffs of M

1. A predicate (capital letter) followed by a constant or variable (lower-case letter) is a wff.

2. If α is a wff, so are

- (∃x)α, (∃y)α, (∃z)α, (∃w)α, (∃v)α
- (x)α, (y)α, (z)α, (w)α, (v)α

3. If α is a wff, so is ~α.

4. If α and β are wffs, then so are:

- (α · β)
- (α ∨ β)
- (α ⊃ β)
- (α ≡ β)

By convention, you may drop the outermost brackets.

5. These are the only ways to make wffs.

A Few More Terms (1/2)

- A wff constructed only using rule 1 is called an **atomic formula**.
  - Pa
  - Qt
- A wff that is part of another wff is called a **subformula**.
  - In '(Pa • Qb) ⊃ (∃x)Rx', the following are all subformulae:
    - Pa
    - Qb
    - Rx
    - (∃x)Rx
    - Pa • Qb
- Wffs that contain at least one unbound variable are called **open sentences**.
  - Ax
  - (x)(Px ⊃ Qx) ⊃ Rz

A Few More Terms (2/2)

- If a wff has no free variables, it is a **closed sentence**, and expresses a **statement**, or **proposition**.
  - (y)[(Py • Qy) ⊃ (Ra ∨ Sa)]
  - (∃x)(Px • Qx) ∨ (y)(Ay ⊃ By)
- Hint: Translations into **M** should yield closed sentences.
- Quantifiers and connectives are called **operators**, or logical operators.
  - Atomic formulas lack operators.
  - The last operator added according to the formation rules is called the **main operator**.