Philosophy 240: Symbolic Logic
Fall 2010
Mondays, Wednesdays, Fridays: 9am - 9:50am

Hamilton College
Russell Marcus
rmarcus1@hamilton.edu

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Translation, Predicate Logic I (§8.1)

I. Introduction

Consider the following argument:

> All philosophers are happy.
> Emily is a philosopher.
> So, Emily is happy.

The conclusion follows logically from the premises, but not when regimented into propositional logic:

> P
> Q / R

Propositional logic is thus insufficient to derive all logical consequences.
Propositional logic represents entailments among sentences.
The entailments in the argument above are within the simple sentences.
We need a logic that explores entailments inside the sentences.
Predicate logic does just that.

In Propositional Logic, we have the following elements:
- Simple terms for statements, capital English letters
- Five connectives
- Punctuation (brackets)

In Predicate Logic, we have the following elements:
- Complex Terms for statements, made of objects and predicates
- Quantifiers
- The same five connectives as in propositional logic.
- The same punctuation as in propositional logic.

II. Objects and Predicates

We represent objects using lower case letters.
- 'a, b, c,...u' stand for specific objects, and are called constants.
- 'v, w, x, y, z' are used as variables.

We represent properties of objects using capital letters, called predicates.
These stand for properties of the objects, and are placed in front of the object letters.

> Pa: means object a has property P, and is said "P of a"
> Pe: Emily is a philosopher
> He: Emily is happy

III. Exercises A. Translate each sentence into predicate logic.

1. Alice is clever.
2. Bobby works hard.
3. Chuck plays tennis regularly.
4. Dan will see Erika on Tuesday at noon in the gym.

IV. Quantifiers

Consider: ‘All philosophers are happy’
The subject of this sentence is not a specific philosopher, no specific object.
Similarly for, ‘Something is made in the USA’
There is no a specific thing to which the sentence refers.
For sentences like these, we use quantifiers.

There are two kinds of quantifiers.
1. Existential quantifiers: (∃x), (∃y), (∃z), (∃w), (∃v)
Existential quantifiers are used with any of the following expressions:

> There exists a thing, such that
> For some thing
> There is a thing
> For at least one thing
> Something

2. Universal quantifiers: (x), (y), (z), (w), (v)
Universal quantifiers are used with:

> For all x
> Everything

Some terms, like ‘anything’, can indicate either quantifier, depending on the context:
In ‘If anything is missing, you’ll be sorry’, we use an existential quantifier.
In ‘Anything goes’, we use a universal quantifier.

Examples of simple translations using quantifiers:

> Something is made in the USA: (∃x)Ux
> Everything is made in the USA: (x)Ux
> Nothing is made in the USA: (x)~Ux or ~(∃x)Ux

Note that all statements with quantifiers and negations can be translated in at least two different ways.

Most sentences get translated as (at least) two predicates:
One is used for the subject of the sentence.
One is used for the attribute of the sentence.
Universals tend to use conditionals (as main connective) to separate the subject from the attribute.
Existentials usually use conjunctions between the subject predicate and the attribute predicate.

More sample translations:

All persons are mortal:	$(x)(Px \supset Mx)$		
Some actors are vain:	$(\exists x)(Ax \cdot Vx)$		
Some gods aren't mortal:	$(\exists x)(Gx \cdot \sim Mx)$		
No frogs are people:	$(x)(Fx \supset \sim Px)$	or	$\sim(\exists x)(Fx \cdot Px)$

V. Exercises B. Translate each sentence into predicate logic.

1. All roads lead to Rome. (Rx, Lx)
2. Beasts eat their young. (Bx, Ex)
3. Everything worthwhile requires effort. (Wx, Rx)
4. Some jellybeans are black. (Jx, Bx)
5. Some jellybeans are not black.

VI. Propositions with more than two predicates

Examples with more than one predicate in the subject part:

Some wooden desks are uncomfortable:	$(\exists x)[(Wx \cdot Dx) \cdot \sim Cx]$
All wooden desks are uncomfortable:	$(x)[(Wx \cdot Dx) \supset \sim Cx]$

Examples with more than one predicate in the attribute part:

Many applicants are untrained or inexperienced:	$(\exists x)[Ax \cdot (\sim Tx \vee \sim Ex)]$
All applicants are untrained or inexperienced:	$(x)[Ax \supset (\sim Tx \vee \sim Ex)]$

Start by asking whether the sentence is universal or existential.
Then, it is often helpful to think of a sentence in terms of the ordinary rules of subject-predicate grammar.
What are we talking about?
What are we saying about it?
The 'what we are talking about' goes as the antecedent in the universally quantified statement, and as the first conjunct in the existentially quantified statement.
The 'what we are saying about it' goes as the consequent or as the second conjunct.

VII. Only

'Only' usually indicates a universal, but translations using 'only' can be tricky.
Consider 'only men have been presidents'.
If something has been a president, it must have been a man: all presidents have been men.
Thus, this sentence should be equivalent to 'all presidents have been men'.
That is, 'only Ps are Qs' is taken to be logically equivalent to 'all Qs are Ps'.
Thus, in some simple cases, we can just invert the antecedent and consequent of a parallel sentence that uses 'all'.

All men have been presidents:	$(x)(Mx \supset Px)$
Only men have been presidents:	$(x)(Px \supset Mx)$

When sentences using only have more than two predicates, they may be grammatically ambiguous.

Only famous men have been presidents:	$(x)[(Px \supset (Mx \cdot Fx)]$	
or:	$(x)[(Px \cdot Mx) \supset Fx]$	?

The two statements are not logically equivalent to it.
The first says that if something is a president, then it must be a famous man.
The second says that if something is a male president, then it must be famous.
Imagine a situation in which there are both men and women presidents.
Of the women presidents, some have been famous, and some have been obscure.
But, all of the men who have been president have been famous.
In such a case, we would favor the second regimentation.
But, if we take 'president' to refer to presidents of the United States, the former regimentation is better.
Thus, extra-logical information, and not the grammar of the sentence, favors the given version.

In contrast, consider:

Only intelligent students understand Kant:	$(x)[Ux \supset (Ix \bullet Sx)]$	
or:	$(x)[(Ux \bullet Sx) \supset Ix)]$	?

In this case, we favor the second interpretation, rather than the first.
The first version says that anyone who understands Kant must be an intelligent student.
It follows from that regimentation that I don't understand Kant, since I am no longer a student.
(Well, I'm a student in some wide metaphorical sense, but not in a literal one.)
In contrast, the second regimentation says that any student who understands Kant is intelligent, which seems a reasonable thing to say.
Again, extra-logical information, rather than grammar, is our guide.
We need not assume that everything that is said is reasonable; that's surely a false assumption.
But, it is customary to charitably presume reasonableness unless we have reason not to.

VII. Propositions with more than one quantifier

If anything is damaged, then everyone in the house complains:	$(\exists x)Dx \supset (x)[(Ix \cdot Px) \supset Cx]$
Either all the gears are broken, or a cylinder is missing:	$(x)(Gx \supset Bx) \vee (\exists x)(Cx \cdot Mx)$

VII. Exercises C. Translate each sentence into predicate logic.

1. Some jellybeans are tasty. (Jx, Tx)
2. Some black jellybeans are tasty. (Jx, Bx, Tx)
3. No green jellybeans are tasty. (Gx, Jx, Tx)
4. Some politicians are wealthy and educated. (Px, Wx, Ex)
5. All wealthy politicians are electable. (Wx, Px, Ex)
6. If all jellybeans are black then no jellybeans are red. (Jx, Bx, Rx)
7. If everything is physical then there are no ghosts. (Px, Gx)
8. Some one walked the dog, but no one washed the dishes. (Px, Wx, Dx)
9. Everyone can go home only if all the work is done. (Px, Gx, Wx, Dx)

VIII. Solutions

Answers to Exercises A:
1. Ca
2. Wb
3. Pc
4. Sd

Answers to Exercises B:
1) (x)(Rx ⊃ Lx)
2) (x)(Bx ⊃ Ex)
3) (x)(Wx ⊃ Rx)
4) (∃x)(Jx · Bx)
5) (∃x)(Jx · ~Bx)

Answers to Exercises C:
1) (∃x)(Jx · Tx)
2) (∃x)[(Bx · Jx) · Tx)]
3) (x)[(Gx · Jx) ⊃ ~Tx] or ~(∃x)[(Gx · Jx) · Tx]
4) (∃x)[Px · (Wx · Ex)]
5) (x)[(Wx · Px) ⊃ Ex]
6) (x)(Jx ⊃ Bx) ⊃ (x)(Jx ⊃ ~Rx) or (x)(Jx ⊃ Bx) ⊃ ~(∃x)(Jx · Rx)
7) (x)Px ⊃ ~(∃x)Gx
8) (∃x)(Px · Wx) · ~(∃x)(Px · Dx)
9) (x)(Px ⊃ Gx) ⊃ (x)(Wx ⊃ Dx)