Philosophy 240
Symbolic Logic

Russell Marcus
Hamilton College

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Translation into Predicate Logic I (§8.1)

Propositional Logic and Predicate Logic

- In Propositional Logic, we have the following elements:
  - Simple terms for statements, capital English letters
  - Five connectives
  - Punctuation (brackets)
- In Predicate Logic, we have the following elements:
  - Complex Terms for statements, made of objects and predicates
  - Quantifiers
  - The same five connectives
  - The same punctuation

Objects and Predicates

- We represent objects using lower case letters.
  - ‘a, b, c,...u’ stand for specific objects, and are called constants.
  - ‘v, w, x, y, z’ are used as variables.
- We represent properties of objects using capital letters, called predicates.
  - Pa: means object a has property P, and can be read “P of a”
  - Pe: Emily is a philosopher
  - He: Emily is happy

Exercises

1. Alice is clever.
2. Bobby works hard.
3. Chuck plays tennis regularly.
4. Dan will see Erika on Tuesday at noon in the gym.

Two Kinds of Quantifiers

- Existential quantifiers: (∃x), (∃y), (∃z), (∃w), (∃v)
  - There exists a thing, such that
  - For some thing
  - There is a thing
  - For at least one thing
  - Something
- Universal quantifiers: (x), (y), (z), (w), (v)
  - For all x
  - Everything
- The amibguity of ‘anything’
  - In ‘If anything is missing, you’ll be sorry’, we use an existential quantifier.
  - In ‘Anything goes’, we use a universal quantifier.

Translations Using Quantifiers

- One predicate
  - Something is made in the USA.
    $(\exists x)Ux$
  - Everything is made in the USA.
    $(x)Ux$
  - Nothing is made in the USA.
    $(x)\sim Ux$
    or
    $\sim(\exists x)Ux$
- More than one predicate:
  - All persons are mortal.
    $(x)(Px \supset Mx)$
  - Some actors are vain.
    $(\exists x)(Ax \cdot Vx)$
  - Some gods aren't mortal.
    $(\exists x)(Gx \cdot \sim Mx)$
  - No frogs are people.
    $(x)(Fx \supset \sim Px)$or$\sim(\exists x)(Fx \cdot Px)$

Exercises

1. All roads lead to Rome. (Rx, Lx)
2. Beasts eat their young. (Bx, Ex)
3. Everything worthwhile requires effort. (Wx, Rx)
4. Some jellybeans are black. (Jx, Bx)
5. Some jellybeans are not black.

Propositions With More Than Two Predicates

- More than one predicate in the subject:
  - Some wooden desks are uncomfortable.
    $(\exists x)[(Wx \cdot Dx) \cdot \sim Cx]$
  - All wooden desks are uncomfortable
    $(x)[(Wx \cdot Dx) \supset \sim Cx]$
- More than one predicate in the attribute:
  - Many applicants are untrained or inexperienced
    $(\exists x)[Ax \cdot (\sim Tx \vee \sim Ex)]$
  - All applicants are untrained or inexperienced
    $(x)[Ax \supset (\sim Tx \vee \sim Ex)]$

Only

- Only men have been presidents.
  - If something has been a president, it must have been a man.
  - All presidents have been men.
- ‘Only Ps are Qs’ is logically equivalent to ‘all Qs are Ps’.
  - All men have been presidents.
    $(x)(Mx \supset Px)$
  - Only men have been presidents.
    $(x)(Px \supset Mx)$
- More than two predicates
  - Only famous men have been presidents.
    $(x)[(Px \supset (Mx \cdot Fx)]$
    or
    $(x)[(Px \cdot Mx) \supset Fx]$?
  - Only intelligent students understand Kant:
    $(x)[Ux \supset (Ix \bullet Sx)]$
    or
    $(x)[(Ux \bullet Sx) \supset Ix)]$?

More than One Quantifier

- If anything is damaged, then everyone in the house complains.
  - $(\exists x)Dx \supset (x)[(Ix \cdot Px) \supset Cx]$
- Either all the gears are broken, or a cylinder is missing.
  - $(x)(Gx \supset Bx) \vee (\exists x)(Cx \cdot Mx)$

Exercises

1. Some jellybeans are tasty. (Jx, Tx)
2. Some black jellybeans are tasty. (Jx, Bx, Tx)
3. No green jellybeans are tasty. (Gx, Jx, Tx)
4. Some politicians are wealthy and educated. (Px, Wx, Ex)
5. All wealthy politicians are electable. (Wx, Px, Ex)
6. If all jellybeans are black then no jellybeans are red. (Jx, Bx, Rx)
7. If everything is physical then there are no ghosts. (Px, Gx)
8. Some one walked the dog, but no one washed the dishes. (Px, Wx, Dx)
9. Everyone can go home only if all the work is done. (Px, Gx, Wx, Dx)