Philosophy 240: Symbolic Logic Hamilton College
Fall 2010 Russell Marcus
Mondays, Wednesdays, Fridays: 9am - 9:50am rmarcus1@hamilton.edu

Class 2 - August 30
Translation Using Propositional Logic (§6.1, §1.2)

I. Logical Connectives

Natural languages, like English, and many formal languages, have a finite stock of simple, or atomic, sentences and an infinite number of grammatically correct sentences.
To produce new, more complex sentences from simple ones, we use what the grammarian calls conjunctions.
The principle by which we can compose longer sentences from shorter ones is called compositionality.
In logic, we reserve the term 'conjunction' for one of the various logical connectives.
The system of logic that we will study uses five connectives:

Negation: ~
Conjunction: ·
Disjunction: ∨
Material Implication: ⊃
Biconditional: ≡

What follows is a more detailed explication of each of our five connectives.

Negation
Some English indicators of negation: Not, it is not the case that p, p is not true, it is false that p

Examples:
John will take the train: J
John won't take the train: ~J
It's not the case that John will take the train: ~J
John takes the train...not!: ~J

In symbols, all of the following are negations:
~R
~(P · Q)
~{[(A ∨ B) ⊃ C] · ~D}

Conjunction
Some English indicators of conjunction: and, but, also, however, yet, still, moreover, although, nevertheless, both.

Examples:
Angelina walks the dog and Brad cleans the floors: A · B
Although Angelina walks the dog, Brad cleans the floors: A · B
Bob and Ray are comedians: B · R
Carolyn is nice, but Emily is really nice: C · E

In symbols, all of the following are conjunctions:

$P \cdot {\sim}Q$

$(A \supset B) \cdot (B \supset A)$

$(P \vee {\sim}Q) \cdot {\sim}[P \equiv (Q \cdot R)]$

Disjunction

Some English indicators of disjunction: or, either, unless

Examples:

Either Paco makes the Website, or Matt does: $P \vee M$

Jared or Rene will go to the party: $J \vee R$

Justin doesn't feed the kids unless Carolyn asks him to: $J \vee C$

In symbols, all of the following are conjunctions:

${\sim}P \vee Q$

$(A \supset B) \vee (B \supset A)$

$(P \vee {\sim}Q) \vee {\sim}[P \equiv (Q \cdot R)]$

We'll discuss 'unless' in more detail after we are familiar with truth conditions.

The Conditional

Some English indicators of a conditional: if, only if, only when, is a necessary condition for, is a sufficient condition for, implies, entails, provided that, given that, on the condition that, in case.

The conditional is also called 'material implication', or just 'implication'.
In '$A \supset B$', A is called the antecedent, B is called the consequent.
Here are some examples, using 'A' to stand for 'you join me' and 'B' to stand for 'I go to the movies'.

1. If you join me, then I go to the movies.	1. If A then B	1. $A \supset B$
2. You join me if I go to the movies.	2. If B then A	2. $B \supset A$
3. You join me only if (only when) I go to the movies.	3. A only if (only when) B	3. $A \supset B$
4. Your joining me is a necessary condition for my going.	4. A is necessary for B	4. $B \supset A$
5. Your joining me is a sufficient condition for my going.	5. A is sufficient for B	5. $A \supset B$
6. A necessary condition of your joining me is my going.	6. B is necessary for A	6. $A \supset B$
7. A sufficient condition for your joining me is my going.	7. B is sufficient for A	7. $B \supset A$
8. Your joining me entails (implies) that I go to the movies.	8. A entails (implies) B	8. $A \supset B$
9. You join me given (provided, on the condition) that I go.	9. A given B	9. $B \supset A$

Note that necessary conditions are consequents, while sufficient conditions are antecedents.
If A is necessary for B, then if B is true, we can infer that A must also be true.
We use the mnemonic 'SUN' to remember this, changing the 'U' to a '$\supset$' we get '$S \supset N$'.

In symbols, all of the following are conditionals:

${\sim}P \supset Q$

$(A \supset B) \supset (B \supset A)$

$(P \vee {\sim}Q) \supset {\sim}[P \equiv (Q \cdot R)]$

The Biconditional

Some English indicators of a biconditional: if and only if, is a necessary and sufficient condition for, just in case.

The biconditional is short for '$(A \supset B) \cdot (B \supset A)$', to which we will return, once we are familiar with truth conditions.

An example:

You'll be successful just in case you work hard and are lucky: $S \equiv L$

In symbols, all of the following are biconditionals:

$\sim P \equiv Q$
$(A \supset B) \equiv (B \supset A)$
$(P \vee \sim Q) \equiv \sim[P \equiv (Q \cdot R)]$

II. Exercises A. Translate to Propositional Logic, using obvious letters for the legend:

1. Alvin doesn't like sports.
2. Bert and Ernie are muppets.
3. Claudia wants to surf or snorkel.
4. Dogs bite just in case they are startled.
5. Everyone loves logic, or not.
6. If Flora wants candy, Geronimo will get her some.
7. Harold is generous unless his wife is listening.
8. Toyota opens a new plant only if Honda initiates an ad campaign.

III. Ambiguous cases

Consider: 'You may have salad or potatoes and carrots.'
Do we translate this as '$(S \vee P) \vee C$'?
Or as '$S \vee (P \vee C)$'?
Look to commas and semicolons, and translate accordingly, using parentheses:
You may have salad, or potatoes and carrots: $S \vee (P \vee C)$
You may have salad or potatoes, and carrots: $(S \vee P) \vee C$
Commas are almost always located at the main connective.

IV. Wffs and Main Connectives

A wff is a 'well-formed formula' and is pronounced "woof", as if you are barking.
Compare:'baker' and 'aebkr'.
One is a word and the other is not a word.
We call statements of logic which are constructed properly 'wffs'.
These are wffs:

$P \cdot Q$
$(\sim P \vee Q) \supset \sim R$

These are not wffs:

> ·P Q
> PqvR~

Formation rules for wffs

1. A single capital English letter is a wff.
2. If α is a wff, so is ~α.
3. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
   By convention, you may drop the outermost brackets.
4. These are the only ways to make wffs.

Main connectives
The last connective added according to the formation rules is called the main connective.

Analyze: (~M ⊃ P) · (~N ⊃ Q)

> 'M', 'P', 'N', and 'Q' are all wffs, by rule 1.
> '~M' and '~N' are wffs by rule 2.
> '(~M ⊃ P)' and '(~N ⊃ Q)' are then wffs by rule 3.
> Finally, the whole formula is a wff also by rule 3, and the convention of dropping the outermost brackets.

V. Exercises B. Are the following formulas wffs? If so, which connective is the main connective?

1. (P ∨ Q) ⊃ ~R
2. ~X(Y ∨ Z)
3. (S ∨ T · U) ⊃ S
4. ~(G ⊃ H)
5. ~{(P ⊃ Q) · [P ≡ ~(Q ∨ R)]}
6. ~[A · (B ∨ C)] ≡ [(A · B) ∨ (A · C)]
7. [(D · E) ∨ F] · G

VI. Exercises C. Translate these sentences into propositional logic, using obvious letters:

1. Ford introduces a new model and either Chrysler raises prices or General Motors changes colors.
2. Both Toyota does not open a new plant and Ford does not introduce a new model.
3. Honda initiates an ad campaign if and only if Chrysler raises prices.
4. Either Saab increases salaries and Toyota opens a new plant or Honda initiates an ad campaign and General Motors changes colors.

5. Toyota's opening a new plant is a necessary condition for General Motors' changing colors, and Ford's introducing a new model is a sufficient condition for Chrysler's raising prices.
6. If Saab increases salaries, then if Toyota opens a new plant, then Honda initiates an ad campaign.
7. Audi lays off workers; however, if Chrysler raises prices then either General Motors does not change colors or Ford does not introduce a new model.

VII. Translation from logic to English

Use the following key:

A: Bob owns an Audi
B: Bob owns a BMW
C: Bob owns a car
D: Bob drives
E: Ethel owns a BMW
F: Fred owns a BMW

Translate together

$B \cdot \sim(E \vee F)$	Bob owns a BMW, but neither Fred nor Ethel do.
$D \equiv C$	Bob owns a car just in case he drives

VIII. Exercises D. Using the above key, translate each of the following sentences into English.

1. $C \supset (A \vee B)$
2. $E \cdot \sim F$
3. $\sim A \supset (\sim D \vee B)$
4. $\sim (A \vee B) \supset \sim C$
5. $\sim(A \cdot B) \cdot C$
6. $(F \cdot E) \equiv \sim B$

IX. Solutions

Answers to Exercises A (Note that alternative letters are possible):

1. $\sim A$
2. $B \cdot E$
3. $F \vee L$
4. $B \equiv S$
5. $L \vee \sim L$
6. $F \supset G$
7. $G \vee L$
8. $T \supset H$

Answers to Exercises B:

1. Yes, ⊃
2. No
3. No
4. Yes, ~
5. Yes, ~
6. Yes, ≡
7. Yes, ·

Answers to Exercises C:

1. F · (C ∨ G)
2. ~T · ~F
3. H ≡ C
4. (S · T) ∨ (H · G)
5. (G ⊃ T) · (F ⊃ C)
6. S ⊃ (T ⊃ H)
7. A · [C ⊃ (~G ∨ ~F)]

Answers to Exercises D:

1. If Bob owns a car, then it's either an Audi or a BMW
2. Ethel owns a BMW, but Fred doesn't
3. If Bob doesn't own an Audi, then either he doesn't drive, or he owns a BMW
4. If Bob owns neither an Audi nor a BMW, then he doesn't own a car.
5. Bob doesn't own both and Audi an a BMW, but he owns a car.
6. Fred and Ethel own BMW's if, and only if, Bob doesn't.

Note: alternate formulations are possible.