Philosophy 240: Symbolic Logic | Hamilton College
Fall 2010 | Russell Marcus
Mondays, Wednesdays, Fridays: 9am - 9:50am | rmarcus1@hamilton.edu

Class 1 - August 27
Arguments; Validity and Soundness (§1.1, §1.4)

I. Introduction

Handouts: syllabus, rules sheet, paper assignment, course bibliography
The course website is:
http://www.thatmarcusfamily.org/philosophy/Course_Websites/Logic_F10/Course_Home.html
Our grader is Megha Hoon, mhoon@hamilton.edu
Peer tutoring is available.

The work in this course will become increasingly difficult.
Good habits are important.
Everyone must hand in the first six homework sets.
After the first test, you are only required to hand in homework if you received lower than 85% on the most recent test.
There will be no make-up tests.
The final exam the lot of difficult material from the end of the course; having an 'A' going in is no guarantee of having an A at the end.

II. Defining 'Logic'

Consider the following pair of definitions:

> A: Logic is the study of argument.
> B: Arguments are what logic studies.

There is a circularity here, which makes the definitions unhelpful.
This circularity is a formal result.
We study formal results, in logic.

Now, replace B., above, with:

> B': An argument is a set of statements, called premises, intended to establish a specific point, called the conclusion.

B' is a better definition.
It is not circular.
It reduces a term to be defined to simpler ones.

Here is another definition: a 'proposition', or a 'statement', is a declarative sentence that has a truth value.
We will consider only two truth values: true and false.
There are logics with more than two truth values.
The most interesting logics have three, or infinitely many.

In this course, we will mostly study two-valued, or bivalent, logic.
But, we will look at three-valued logics a little bit.

Pairing A and B', we think about logic as the rules of what follows from what, of consequences from assumptions.
Logic is used in philosophical arguments, and in scientific reasoning.
An alternative description thus characterizes logic as the rules of reasoning.
Before the nineteenth century, logic was considered to be the rules of thought.
Taking logic to be the science of reasoning raises a question:
Is logic descriptive, representing how we actually reason, or is it prescriptive, setting out rules for proper reasoning?
Before we can start to answer this question, we have to see what logic looks like, at least a bit.

III. A Short History of Logic

Aristotle, who lived in the fourth century B.C.E., famously described some fundamental logical rules, called categorical syllogisms.
The categorical syllogisms described relations among four kinds of statements, known since the early middle-ages as A, E, I, and I.

A. All Fs are Gs.
E. No Fs are Gs.
I. Some Fs are Gs.
O. Some Fs are not Gs.

In categorical logic, the fundamental elements are terms, portions of assertions.
We will look at the modern version of term logic, called predicate or quantificational logic, in the second half of the course.

In the third century B.C.E., the stoic philosopher Chrysippus developed a propositional logic, in which the fundamental elements are complete assertions.
Some assertions are simple, others are complex.
Complex assertions are composed of simple assertions combined according to logical rules.
In the first half of the course, we will look at the rules of propositional logic.

Through the middle ages, while there were some major advances in logic, the structure of the discipline was generally stable.
After the scientific revolution, philosophers started paying more attention to human psychological capacities.
This focus, which we can see in Descartes, Locke, and Hume culminated in the late-eighteenth century work of Kant, and the early nineteenth-century work of Hegel.
Kant’s logic was essentially a description of how human beings create their experiences by imposing, a priori, conceptual categories on an unstructured manifold given in sensation.
Logic had become the description of human psychology, instead of the rules of logical consequence.

In addition, in the nineteenth century, several developments led mathematicians to worry about logical entailments.

For nearly two hundred years, mathematicians had worked with the calculus of Newton and Leibniz.
The calculus allowed mathematicians to find the area under a curve by dividing the area into infinitely many infinitely small areas.
Working with infinity, both small and large, seemed indefensible, even if the resulting calculations were successful.
To make matters worse, Cantor, in the mid-nineteenth century discovered a proof that there are different sizes of infinity, indeed there are infinitely many difference sizes of infinity.
Cantor's claim struck many mathematicians as absurd, but they could not find a flaw in his logic.

Similarly, consider Euclid's controversial parallel postulate.
According to the parallel postulate, given a line, and a point outside that line, there is one and only line which passes through the point parallel to the given line.
(Actually, that formulation of Euclid's fifth postulate is called Playfair's postulate; Euclid worked with an equivalent but different formulation.)
For centuries, Euclid's postulate seemed too controversial to be adopted as an unproven axiom, yet it resisted proof from the other axioms.
Research into the parallel postulate had led, by the early nineteenth century, not to its proof, but to the realization that one could develop two different consistent systems of geometry by denying the postulate.
If one adopts the claim that there are no lines through the given point parallel to the given line, the geometry of spheres is created.
If one adopts the claim that there are more than one parallel line, one creates a system of hyperbolic geometry.
In the twentieth century, hyperbolic geometry was shown not only to be consistent; it turns out to be the correct geometry for space-time, according to the theory of relativity.
Again, mathematicians considering the counter-intuitive results of non-Euclidean geometries worried that the laws of logical consequence were being flouted.
Mathematicians and philosophers began to think more carefully about the notion of logical consequence.

In 1879, Gottlob Frege published the *Begriffsschrift*, a mathematical logical calculus which subsumed both the term logic of Aristotle and the propositional logic of the stoics.
Further, Frege's logic extended and refined the rules of logic, generalizing results.
Frege's work, while not immediately recognized as revolutionary, became the foundation for fifty years of intense research in the logical foundations of mathematics and reasoning generally.
The culmination of this flurry of research came in the early 1930s with Alfred Tarski's work on truth and Kurt Gödel's incompleteness theorems.
Frege's logic, in a neater and more perspicuous form, is the focus of this course.

IV. Separating Premises from Conclusions

Our first task is to analyze arguments, indicating their structures and separating premises from conclusions.
Consider the following argument:

> We may conclude that eating meat is wrong. This may be inferred from the fact that we must kill to get meat. And killing is wrong.

The conclusion is: 'Eating meat is wrong.'

The premises are: ‘We must kill to get meat. Killing is wrong.’
Note the elimination of certain words: these are indicators.
When formally representing arguments, we omit indicators.

Here are some conclusion indicators:

therefore
we may conclude that
we may infer that
entails that
hence
thus
consequently
so
it follows that
implies that
as a result.

Here are some premise indicators:

since
because
for
in that
may be inferred from
given that
seeing that
for the reason that
inasmuch as
owing to

‘And’ often indicates the presence of an additional premise.
Natural language is inexact, and non-formulaic.
Not all sentences will contain indicators.
You will have to judge from their content which propositions are premises and which are conclusions.
The best way to determine premises and conclusions is to determine what the main point is, and then look to see what supports that point.

Some arguments contain irrelevant, extraneous information.
Some arguments contain implicit information; these are called ‘enthymemes’.

We can represent the argument above in the following manner:

P1: We must kill to get meat.
P2: Killing is wrong.
C: Eating meat is wrong.

The order of the premises is unimportant.
The number of premises is unimportant: you may combine or separate premises, at times.
Sometimes, a sentence may contain both a premise and a conclusion, and so must be divided.

V. Exercises A. Represent the following arguments in premise/conclusion form.

1. The psychological impact and crisis created by the birth of a defective infant is devastating. Not only is the mother denied the normal tension release from the stress of pregnancy, but both parents feel a crushing blow to their dignity , self-esteem, and self-confidence. In a very short time, they feel grief for the loss of the normal, expected child, anger at fate, numbness, disgust, waves of helplessness and disbelief.

2. Neither a borrower nor a lender be,
For loan oft loses both itself and friend,
And borrowing dulls the edge of husbandry.

3. If a piece of information is not "job relevant," then the employer is not entitled qua employer to know it. Consequently, since sexual practices, political beliefs, associational activities, etc., are not part of the descriptions of most jobs, that is, since they do not directly affect one's job performance, they are not legitimate information for an employer to know in the determination of the hiring of a job applicant.

VI. Validity and Soundness

Consider the following three arguments. Are they good?

1. All persons are mortal.
   Socrates is a person.
   ∴ Socrates is mortal.

2. All men are fish.
   Joe is a man.
   ∴ Joe is a fish.

3. All Toyotas are cars.
   I own a car.
   ∴ I own a Toyota.

Argument 1 is good.
Arguments 2 and 3 are both bad, but for different reasons.
Argument 3 is invalid.
Argument 2 is valid, but unsound.

The validity of an argument depends on its form.
An argument is valid if the conclusion follows logically from the premises.
Certain forms are valid.
Certain forms are invalid
The soundness of a valid argument depends on truth of its premises.
A valid argument is sound if its premises are true.
Only valid arguments can be sound.

Here is the most important sentence of this course:

> In deductive logic, if the form of an argument is valid and the premises are all true, then the conclusion must be true.

In invalid arguments, the premises can be true at the same time that the conclusion is false, though all can be true.
Validity is independent of truth.
Validity is related to possibility, while soundness is related to truth.

VII. Exercises B. Are the following valid? If so, are they sound?

1. If it snows more than two feet, there will be no classes at Hamilton. It snowed more than two feet last Monday. Therefore, there were no classes at Hamilton.

2. The Mets are a professional baseball team. Professional baseball teams are sports businesses. So, the Mets are a sports business.

3. If police departments improve their effectiveness, crime rates go down. Crime rates have gone down. So, police departments have improved their effectiveness.

4. Since the sun is pink, and made of cheese, it follows that some cheese is pink.

5. Some cars are green. Some cars are Toyotas. So, some cars are green Toyotas.

6. All great singers have strong voices. Celine Dion does not have a strong voice. So Celine Dion is not a great singer.

VIII. The form of an argument

Consider each of the following arguments:

1. Either the stock market will rise or unemployment will go up.
The market won't rise.
So, unemployment will increase.

2. You will get either rice or beans.
You don't get the rice.
So, you'll have the beans.

3. The square root of two is either rational or irrational.
It's not rational.
So, it's irrational.

They have the same form:

Either p or q
not-p
So, q.

This form is called ‘Disjunctive Syllogism’
We’ll study it later.
Just as an architect, when building a building, looks only at the essential structures, so a logician looks only at the form of an argument.
‘p’ and ‘q’, above, are like variables, standing for statements;
‘either p or q’ is a compound sentence, made of simple ones

The language of propositional logic uses capital letters to stand for simple, positive propositions.
Simple propositions are often of subject-predicate form, but not necessarily.
They are the shortest examples of statements; they can not be decomposed further in propositional logic.
In predicate logic, we go beneath the surface, at end of term.

IX. Solutions

Answers to Exercise A:

1. Premise 1: Not only is the mother denied the normal tension release from the stress of pregnancy, but both parents feel a crushing blow to their dignity , self-esteem, and self-confidence.
   Premise 2: In a very short time, they feel grief for the loss of the normal, expected child, anger at fate, numbness, disgust, waves of helplessness and disbelief.
   Conclusion The psychological impact and crisis created by the birth of a defective infant is devastating.

2. Premise 1: Loan oft loses both itself and friend.
   Premise 2: Borrowing dulls the edge of husbandry.
   Conclusion: Neither a borrower nor a lender be.

3. Premise 1: If a piece of information is not "job relevant," then the employer is not entitled qua employer to know it.
   Premise 2: Sexual practices, political beliefs, associational activities, etc., are not part of the descriptions of most jobs, that is, they do not directly affect one's job performance,
   Conclusion: They are not legitimate information for an employer to know in the determination of the hiring of a job applicant.

Answers to Exercises B:

1. Valid, unsound
2. Valid, sound
3. Invalid
4. Valid, unsound
5. Invalid
6. Valid, unsound