

Solutions to Practice Problems for Test #5

These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

I. Derivations

1.
 

1. $Ab \vee Bc$	
2. $(x)\sim Bx$	$/ (\exists x)Ax$
3. $\sim Bc$	2, UI
4. $Ab$	1,3 Com, DS
5. $(\exists x)Ax$	4, EG

QED

2.
 

1. $(x)(Rx \supset Ox)$	
2. $(\exists x)\sim Ox$	
3. $(x)(\sim Rx \supset Px)$	$/ (\exists x)Px$
4. $\sim Oa$	2, EI
5. $Ra \supset Oa$	1, UI
6. $\sim Ra$	5, 4, MT
7. $\sim Ra \supset Pa$	3, UI
8. $Pa$	7, 6, MP
9. $(\exists x)Px$	8, EG

QED

3.
 

1. $(x)(Fx \supset Gx)$	
2. $(y)(Gy \supset Hy)$	$/ (z)(\sim Hz \supset \sim Fz)$
3. $Fx \supset Gx$	1, UI
4. $Gx \supset Hx$	2, UI
5. $Fx \supset Hx$	3, 4, HS
6. $\sim Hx \supset \sim Fx$	5, Trans
7. $(z)(\sim Hz \supset \sim Fz)$	6, UG

QED

4.
 

1. $(\exists x)(Ax \cdot Bx) \supset (x) Dx$	
2. $\sim Da$	$/ (x)(Ax \supset \sim Bx)$
3. $(\exists x)\sim Dx$	2, EG
4. $\sim (x)Dx$	3, CQ
5. $\sim (\exists x)(Ax \cdot Bx)$	1, 4, MT
6. $(x)\sim (Ax \cdot Bx)$	5, CQ
7. $(x)(\sim Ax \vee \sim Bx)$	6, DM
8. $(x)(Ax \supset \sim Bx)$	7, Impl

QED



- 8.
- |   |                                       |
|---|---------------------------------------|
| 1. $(x)(Ax \supset Bx)$                   |                                       |
| 2. $(x)[Bx \supset (Ax \supset \sim Fx)]$ |                                       |
| 3. $(x)[(\sim Cx \cdot Dx) \supset Fx]$   | $/ (x)[Ax \supset (Cx \vee \sim Dx)]$ |
| 4. $Ax$                                   | ACP                                   |
| 5. $Ax \supset Bx$                        | 1, UI                                 |
| 6. $Bx$                                   | 5, 4, MP                              |
| 7. $Bx \supset (Ax \supset \sim Fx)$      | 2, UI                                 |
| 8. $Ax \supset \sim Fx$                   | 7, 6, MP                              |
| 9. $\sim Fx$                              | 8, 4, MP                              |
| 10. $(\sim Cx \cdot Dx) \supset Fx$       | 3, UI                                 |
| 11. $\sim(\sim Cx \cdot Dx)$              | 10, 9, MT                             |
| 12. $Cx \vee \sim Dx$                     | 11, DM, DN                            |
| 13. $Ax \supset (Cx \vee \sim Dx)$        | 4-12, CP                              |
| 14. $(x)[Ax \supset (Cx \vee \sim Dx)]$   | 13, UG                                |

QED

- 9.
- |   |               |
|---|---------------|
| 1. $(\exists x)Gx \supset (x)(Fx \supset Dx)$ |               |
| 2. $(\exists x)(Gx \cdot \sim Dx)$            | $/ \sim(x)Fx$ |
| 3. $Ga \cdot \sim Da$                         | 2, EI         |
| 4. $Ga$                                       | 3, Simp       |
| 5. $(\exists x)Gx$                            | 4, EG         |
| 6. $(x)(Fx \supset Dx)$                       | 1, 5, MP      |
| 7. $Fa \supset Da$                            | 6, UI         |
| 8. $\sim Da$                                  | 3, Com, Simp  |
| 9. $\sim Fa$                                  | 7, 8, MT      |
| 10. $(\exists x)\sim Fx$                      | 9, EG         |
| 11. $\sim(x)Fx$                               | 10, CQ        |

QED

- 10.
- |   |                   |
|---|-------------------|
| 1. $(\exists x)Qx \supset (x)(Rx \supset Sx)$ |                   |
| 2. $(x)\sim Qx \supset (\exists x)Sx$         |                   |
| 3. $(x) Rx$                                   | $/ (\exists x)Sx$ |
| 4. $\sim(\exists x)Sx$                        | AIP               |
| 5. $\sim(x)\sim Qx$                           | 2, 4, MT          |
| 6. $(\exists x)Qx$                            | 5, CQ             |
| 7. $(x)(Rx \supset Sx)$                       | 1, 6, MP          |
| 8. $Rx \supset Sx$                            | 7, UI             |
| 9. $Rx$                                       | 3, UI             |
| 10. $Sx$                                      | 8, 9, MP          |
| 11. $(\exists x)Sx$                           | 10, EG            |
| 12. $(\exists x)Sx \cdot \sim(\exists x)Sx$   | 11, 4, Conj       |
| 13. $(\exists x)Sx$                           | 4-12, IP, DN      |

QED

## II. Invalidity

1. Invalid in a 1-member universe, where:

Aa: True; Ba: False; Ca: False.

2. Invalid in a 2-member universe, where:

Ea: True; Fa: False; Ga: False;

Eb: True or False; Fb: True; Gb: True

3. Invalid in a 3-member universe, where:

Pa: True; Qa: False; Ra: False

Pb: False; Qb: True; Rb: False

Pc: False; Qc: False; Rc: True or False