Solutions to Homework Handout #6: Functions

I.   1. ~Po • Pf(o)
        2. (x)[Nx ⊃ (∃y)y=f(x)]
        3. (x)(y)[(Ny • Oy • x=f(y)) ⊃ Ex]
        4. (x)(y){(Nx • Ny) ⊃ {Eg(x, y) ⊃ [(Ex • Ey) ∨ (Ox • Oy)]}}
        5. (x)(y){(Nx • Ny) ⊃ {Og(x, y) ⊃ [(Ex • Oy) ∨ (Ox • Ey)]}}
        6. (x)(y)[(Nx • Px • Ny • Py) ⊃ ~Pf(x, y)]
        7. (x)[(Nx • Ex) ⊃ Eg(x)] • (x)[(Nx • Ox) ⊃ Og(x)]
        8. (x)[(Nx • Ex) ⊃ Of(g(x))]
        9. (x)[(Nx • Px • ~x=t) ⊃ Og(t, x)]
        10. (∃x)(∃y)[Nx • Px • Ny • Py • ~x=y • f(x, y)=f(g(x, y))]

II. Sample derivations

        1.   1. (x)[Ax ∨ Bf(x)]   / (x)[Af(x) ∨ Bf(f(x))]
                2. Af(x) ∨ Bf(f(x))   1, UI
                3. (x)[Af(x) ∨ Bf(f(x))]   2, UG
        QED

        2.   1. (x)(y)[f(x)=y ⊃ Cyxc]
                2. ~Cf(a)ac   / f(a)≠b
                    │ 3. f(a)=b   AIP
                    │ 4. (y)[f(a)=y ⊃ Cyac]   UI, 1
                    │ 5. f(a)=b ⊃ Cbac   UI, 4
                    │ 6. Cbac   5, 3, MP
                    │ 7. Cf(a)ac   6, 3, ID
                    │ 8. Cf(a)ac • ~Cf(a)ac   7, 2, Conj
                9. f(a)≠b
        QED

        3.   1. (x){Dx ⊃ (y)[~Exy ≡ Gf(f(y))]}
                2. (x)(Dx • ~Gx)   / (x)Ef(x)f(x)
                3. Df(x) • ~Gf(x)   2, UI
                4. Df(x) ⊃ (y)[~Ef(x)y ≡ Gf(f(y))]   1, UI
                5. Df(x)   3, Simp
                6. (y)[~Ef(x)y ≡ Gf(f(y))]   4, 5, MP
                7. Df(f(f(x))) • ~Gf(f(f(x)))   2, UI
                8. ~Ef(x)f(x) ≡ Gf(f(f(x)))   6, UI
                9. [~Ef(x)f(x) ⊃ Gf(f(f(x)))] • [Gf(f(f(x))) ⊃ ~Ef(x)f(x)]   8, Equiv
                10. ~Ef(x)f(x) ⊃ Gf(f(f(x)))   9, Simp
                11. ~Gf(f(f(x)))   7, Com, Simp
                12. Ef(x)f(x)   10, 11, MT, DN
                13. (x)Ef(x)f(x)   12, UG
        QED