

Sample Solutions to Homework Handout #3 - Practice Problems for Test #3

1. 1. $G \supset H$
 2. $G \supset I$ / $G \supset (H \cdot I)$
 | 3. G ACP
 | 4. H 1, 3, MP
 | 5. I 2, 3, MP
 | 6. $H \cdot I$ 4, 5, Conj
 7. $G \supset (H \cdot I)$ 3-6, CP

QED

2. 1. $J \equiv K$
 2. $\sim K \equiv \sim L$ / $J \equiv L$
 3. $(J \supset K) \cdot (K \supset J)$ 1, Equiv
 4. $(\sim K \supset \sim L) \cdot (\sim L \supset \sim K)$ 2, Equiv
 5. $J \supset K$ 3, Simp
 6. $\sim L \supset \sim K$ 4, Com, Simp
 7. $K \supset L$ 6, Trans
 8. $J \supset L$ 5, 7, HS
 9. $\sim K \supset \sim L$ 4, Simp
 10. $L \supset K$ 9, Trans
 11. $K \supset J$ 3, Com, Simp
 12. $L \supset J$ 10, 11, HS
 13. $(J \supset L) \cdot (L \supset J)$ 8, 12, Conj
 14. $J \equiv L$ 13, Equiv

QED

3. 1. M / $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$
 | 2. N ACP
 | | 3. O ACP
 | | | 4. P ACP
 | | | 5. $M \cdot N$ 1, 2, Conj
 | | | 6. $O \cdot P$ 3, 4, Conj
 | | | 7. $(M \cdot N) \cdot (O \cdot P)$ 5, 6, Conj
 | | | 8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$ 4-7, CP
 | | 9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$ 3-8, CP
 10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ 2-9, CP

QED

4. | 1. R ACP
 | | 2. $R \supset S$ ACP
 | | 3. S 2, 1, MP
 | 4. $(R \supset S) \supset S$ 2-3, CP
 5. $R \supset [(R \supset S) \supset S]$ 1-4, CP

QED

- 5.
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|----------------------------|--------------|
| 1. $D \supset E$ | |
| 2. $E \supset (F \cdot G)$ | |
| 3. $\sim F \vee \sim G$ | $/\sim D$ |
| 4. D | AIP |
| 5. E | 1, 4, MP |
| 6. $F \cdot G$ | 2, 5, MP |
| 7. F | 6, Simp |
| 8. $\sim G$ | 3, 7, DN, DS |
| 9. G | 6, Com, Simp |
| 10. $G \cdot \sim G$ | 9, 8, Conj |
| 11. $\sim D$ | 4-10, IP |

QED

- 6.
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|---|----------------|
| 1. $(P \vee Q) \vee (\sim R \vee S)$ | |
| 2. $\sim Q \cdot (\sim S \cdot \sim P)$ | $/\sim R$ |
| 3. $\sim Q$ | 2, Simp |
| 4. $(Q \vee P) \vee (\sim R \vee S)$ | 1, Com |
| 5. $Q \vee [P \vee (\sim R \vee S)]$ | 4, Assoc |
| 6. $P \vee (\sim R \vee S)$ | 5, 3, DS |
| 7. $\sim S \cdot \sim P$ | 2, Com, Simp |
| 8. $\sim P$ | 7, Com, Simp |
| 9. $\sim R \vee S$ | 6, 8, DS |
| 10. $\sim S$ | 7, Simp |
| 11. $\sim R$ | 9, 10, Com, DS |

QED

- 7.
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|--|--------------|
| 1. $\sim \{A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]\}$ | AIP |
| 2. $\sim A \cdot \sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1, DM |
| 3. $\sim A$ | 2, Simp |
| 4. $\sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 2, Com, Simp |
| 5. $\sim [\sim A \vee (B \cdot C)]$ | 4, Dist |
| 6. $A \cdot \sim (B \cdot C)$ | 5, DM, DN |
| 7. A | 6, Simp |
| 8. $A \cdot \sim A$ | 7, 3, Conj |
| 9. $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1-8, IP, DN |

QED

8. 1. $(A \vee B) \supset \sim(F \cdot D)$
 2. $\sim(A \cdot \sim D)$
 3. $\sim F \supset \sim(C \cdot D)$
 4. $C \vee A$ / $A \equiv \sim C$
 | 5. A ACP
 | 6. $A \vee B$ 5, Add
 | 7. $\sim(F \cdot D)$ 6, 1, 6, MP
 | 8. $\sim F \vee \sim D$ 7, DM
 | 9. $\sim A \vee \sim \sim D$ 2, DM
 | 10. $\sim \sim D$ 9, 5, DM, DS
 | 11. $\sim F$ 8, 10, Com, DS
 | 12. $\sim(C \cdot D)$ 3, 11, MP
 | 13. $\sim C \vee \sim D$ 12, DM
 | 14. $\sim C$ 13, 10, Com, DS
 15. $A \supset \sim C$ 5-14, CP
 16. $\sim C \supset A$ 4, DN, Impl
 17. $(A \supset \sim C) \cdot (\sim C \supset A)$ 15, 16, Conj
 18. $A \equiv \sim C$ 17, Equiv

QED

9. 1. $(A \cdot B) \supset C$
 2. $(F \cdot D) \supset E$
 3. $A \cdot D$
 4. $B \vee F$ / $C \vee E$
 | 5. $\sim(C \vee E)$ AIP
 | 6. $\sim C \cdot \sim E$ 5, DM
 | 7. $\sim C$ 6, Simp
 | 8. $\sim(A \cdot B)$ 1, 7, MT
 | 9. $\sim A \vee \sim B$ 8, DM
 | 10. A 3, Simp
 | 11. $\sim B$ 9, 10, DN, DS
 | 12. F 4, 11, DS
 | 13. D 3, Com, Simp
 | 14. $F \cdot D$ 12, 13, Conj
 | 15. E 2, 14, MP
 | 16. $\sim E$ 6, Com, Simp
 | 17. $E \cdot \sim E$ 15, 16, Conj
 18. $C \vee E$ 5-17, IP, DN

QED

10. 1. $\sim F \supset (G \bullet H)$
 2. $F \supset (I \vee J)$
 3. $\sim [G \bullet (K \bullet \sim J)]$
 4. $K \equiv (L \bullet \sim J)$ / $K \supset I$
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|--|----------------|
| 5. K | ACP |
| 6. $[K \supset (L \bullet \sim J)] \bullet [(L \bullet \sim J) \supset K]$ | 4, Equiv |
| 7. $K \supset (L \bullet \sim J)$ | 6, Simp |
| 8. $L \bullet \sim J$ | 7, 5, MP |
| 9. $\sim J$ | 8, Com, Simp |
| 10. $K \bullet \sim J$ | 5, 9, Conj |
| 11. $\sim G \vee \sim (K \bullet \sim J)$ | 3, DM |
| 12. $\sim (K \bullet \sim J) \vee \sim G$ | 11, Com |
| 13. $\sim G$ | 12, 10, DN, DS |
| 14. $\sim I$ | AIP |
| 15. $\sim I \bullet \sim J$ | 14, 9, Conj |
| 16. $\sim (I \vee J)$ | 15, DM |
| 17. $\sim F$ | 2, 16, MT |
| 18. $G \bullet H$ | 1, 17, MP |
| 19. G | 18, Simp |
| 20. $G \bullet \sim G$ | 19, 13, Conj |
| 21. I | 14-20, IP |
| 22. $K \supset I$ | 5-21, CP |

QED