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Class 42: The Right Logic?
Fisher 153-161

I. Branching quantifiers

We have looked at various extensions of full first-order predicate logic: identity theory, functions, and higher-order quantifiers.
Another extension involves branching quantifiers.
Consider:

1. Some relative of each villager and some relative of each townsman hate each other.

We might regiment 1 as:

2. (x){Vx ⊃ (∃y){Ryx • (z)[Tz ⊃ (∃w)(Rwz • Hyw • Hwy)]}}

2 makes the choice of the townspeople dependent on the choice of the villagers.
Turning it around as in 3 makes the choice of the villagers depend on the choice of the townspeople.

3. (x){Tx ⊃ (∃y){Ryx • (z)[Vz ⊃ (∃w)(Rwz • Hyw • Hwy)]}}

What we really want to do is choose each independently, and then express the relation.
We can pull out the quantifiers, according to the rules of passage, but the unwanted dependency remains.

4. (x)(∃y)(z)(∃w)[(Vx ⊃ Ryx) • (Tz ⊃ Rwz) • Hyw • Hwy]

We can avoid any ordering of the choice by branching the quantifiers:

5. (x)(∃y) :
: [(Vx ⊃ Ryx) • (Tz ⊃ Rwz) • Hyw • Hwy]
(z)(∃w) :

The branching indicates that the choice of values for x and y are completely independent of the choice of values for z and w.
Other sentences naturally take branching quantifiers.

6. Some book by every author is referred to in some essay by every critic.

Standard interpretations of branching quantifiers involve functions.
As I mentioned, functions are too close to mathematics for some logicians.

II. Completeness and a canonic language

The underlying question facing us is whether there is a best version of logic, one which can be taken as canonical.
And, if so, which?
We are looking for a line at which to draw the limits of logic.
On one side of the line would be the true logic.
On the other side of the line would be specific domains, like mathematics, or physics, or metaphysics.

As I have mentioned, Quine, among others, favors a canonical language of first-order logic with identity, which I have called **F**.
Hurley's text, for example, follows Quine's prescription.
Perhaps the most popular argument in favor of drawing the line as Quine and Hurley do relies on the completeness of first-order logic with identity.
First-order logic with identity is provably complete.
Stronger logics, like second-order logic, lose completeness.

When one first hears the term 'completeness', it sounds really sexy.
All that 'completeness' really means is that every wff of first-order logic can be either proven or disproved.
Completeness entails nothing about whether there are sentences, like Leibniz's law, that can not be written in first-order logic.
I have introduced functions, second-order quantifiers, and branching quantifiers in order to expand the ability of logic to represent sentences of English.
I am more interested in what we might call expressive completeness than in the traditional concept of completeness.

III. Other technical virtues of first-order logic

There are reasons other than completeness to favor first-order logic over any of its extensions.
In first-order logic, a variety of definitions of logical truth concur: in terms of logical structure, substitution of sentences or of terms, satisfaction by models, and proof.
These definitions cleave in extensions of first-order logic, as Quine demonstrates in Chapter 4 of *Philosophy of Logic*.

There are other technical virtues of first-order logic.
Every consistent first-order theory has a model.
First-order logic is compact, which means that any set of first-order axioms will be consistent if every finite subset of that set is consistent.
It admits of both upward and downward Löwenheim-Skolem features, which mean that every theory which has an infinite model will have a model of every infinite cardinality (upward) and that every theory which has an infinite model of any cardinality will have a denumerable model (downward).
All of these properties fail in second-order logic; see Mendelson, *Introduction to Mathematical Logic*, p 377.
These properties are the subjects of advanced logic courses.
Some of them, like the Löwenheim-Skolem theorem, are extremely interesting.

IV. Change of logic - change of subject

Fisher discusses one final influential argument in favor of logics like **F**, and against extensions of it.
It is called the change of logic - change of subject argument.
The basic idea of the argument is that in order to disagree with someone, you have to at least agree on what you are disagreeing about.
There has to be some common ground on which you can stand, to argue, or else you are not really disagreeing at all.
Consider two terms, and their definitions:

> $Chair_1$: desk chairs, dining room chairs, and such, but not recliners or bean bag chairs
> $Chair_2$: all $chair_1$ objects, and also recliners and bean bag chairs

Now, consider one person, who uses 'chair' as $chair_1$ and another person who uses 'chair' as $chair_2$.
And imagine them both talking about a bean bag chair.
Person 1 affirms 'that's a chair', while Person 2 denies that sentence.
Since they are using the same term, it looks like they are disagreeing.
But they are not really disagreeing about whether the bean bag chair is a chair.
They are disagreeing about what 'chair' means.
In order to describe the disagreement over 'chair', they have to at least agree on how to describe their disagreement.

If we are considering debates over the correct logic, even claims of what it means to affirm or deny a sentence are under discussion.
Debates over the correct logic seem to be more like the disagreement between $chair_1$ and $chair_2$.
The disputants do not agree on the terms they are using, and so are talking past each other.

Imagine we are linguists, and we are headed to a newly-discovered alien planet.
We have to translate a completely new language into English.
We start by assuming that the aliens obey the rules of logic.
If we were to propose a translation of the alien language on which the aliens often made statements that translated into the form of '$P \bullet {\sim}P$', we would revise our translation.
We take the laws of logic as fundamental.
We use them as common ground on which to base our translations.
If we hypothesize that the native is asserting a contradiction, we take that to be evidence against our translation, rather than evidence against the native's intellectual capacity, for example.

We need logic to serve as a starting point for the translation.
We need common ground even to formulate disagreement.
If we disagree about the right logic, then we have merely changed the subject.

V. The existence of God

Lastly, I'll leave you with a bit of a puzzle which affects even our most austere version of predicate logic, **F**.
Consider the following proof of the existence of God.

| 1. ~(∃x)x=g	Assumption, for indirect proof
| 2. (x)x=x	Principle of identity
| 3. (x)~x=g	1, Change of quantifier rule
| 4. g=g	2, UI
| 5. ~g=g	3, UI
| 6. g=g • ~g=g	4, 5, Conj
6. (∃x) x=g	1-5, Indirect proof

In class, I said that I would let you think about what's wrong with it.
A bit of reflection should convince you that the same argument proves the existence of the tooth fairy.
The problem, I believe, is linked to the presence of constants within the language of first-order logic.
So, it is better to avoid them, when translating.

It is easy enough to avoid constants by merely turning them into predicates.
Consider regimenting the following simple sentence using a constant.

7. Obama voted for Obama.
8. Oo

Alternately, one might attempt to translate 7 without using a constant.

9. (∃x)(Ox • Vx)

This last translation does not commit to a unique Obama, but we can fix it.

10. (∃x)[Ox • (y)(Oy ⊃ y=x) • Vx]

It puzzles me that so many logic texts continue to include constants, when it is clear that their presence leads to absurdity.