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Class 4 - September 4
Philosophy Friday #1: Conditionals
Fisher, pp 106-111

I. Various types of conditional sentences

The standard truth table for the material conditional is based on the logic of the indicative conditional in natural language.
We can distinguish indicative conditionals from others:

A: Indicative conditionals: If the Mets lost, then the Cubs won.
B: Conditional questions: If I like logic, what class should I take next?
C: Conditional commands: If you want to pass this class, do the homework.
D: Conditional prescriptions: If you want a good life, you ought to act virtuously.
E: Subjunctive, or counterfactual, conditionals: If he were offered the bribe, he would take it.

A is the material conditional, as we introduced it in our last class.
B, C, and D are not propositions, so not truth-valuable.
We can, if we wish, parse them truth-functionally by turning them into indicatives, perhaps as follows:

B': If you like logic, then you should take linear algebra next.
C': If you want to pass the class, you do the homework.
D': If you want a good life, you act virtuously

But E and other sentences like it are tricky.
We say that a material conditional is true, unless the antecedent is true and the consequent is false.

P	⊃	Q
T	T	T
T	F	F
F	T	T
F	T	F

Another way to say it is, 'If A then B', means 'Not (A and not-B)'
But consider:

F: If I were to jump out of the window right now, I would fall to the ground.
G: If I were to jump out of the window right now, I would fly to the moon.

Here, intuitively, we'd like to say that F is true and G is false.
On the material interpretation, they are both true, since I am not jumping out of the window right now.
The problem arises from the counterfactual claim in the antecedent.

II. Other options for the truth table

Perhaps the problem is in the way we designed the truth table.
The first two lines, which are not counterfactual, look fine.
Remember the house-painting example, from last class.
There are three alternate possibilities for the third and fourth lines.

Option A

p	⊃	q
T	T	T
T	F	F
F	T	T
F	F	F

Option B

p	⊃	q
T	T	T
T	F	F
F	F	T
F	T	F

Option C

p	⊃	q
T	T	T
T	F	F
F	F	T
F	F	F

Fisher discusses problems with each.

Option A gives the conditional the same truth-value as the consequent.
So, it makes the antecedent irrelevant.
On Option A, 'if pigs fly, then 2+2=4' would be true, but 'if pigs fly, then '2+2=5' would be false.
Fisher uses the example: 'if you run a red light, then you break the law'.

Option B makes the conditional the same as a biconditional.
Again, the conditional seems to have a different role.
Fisher's red-light example works nicely, here.
See Row 3, especially.

Option C is the same as the conjunction!
Again Fisher's example demonstrates the difference between conditionals and conjunctions, especially at Row 3.

So, it looks like we have to stick with the original truth table.

III. Counterfactual conditionals and science

One option is to ignore counterfactual conditionals.
For most of our purposes in this course, we can put them aside.
When regimenting, we could just remember that the truth-functional conditional is not applicable to subjunctives.
But, these counterfactual conditionals are important in science.
Consider:

G: If this salt had been placed in water, it would have dissolved.

G is important because it indicates a dispositional property of salt, one that we use to characterize the substance.
Other dispositional properties, like irritability, flammability, and flexibility, refer to properties interesting to scientists.
Psychological properties, like believing that it is cold outside, are often explained as dispositions to behave, like the disposition to put on a coat or say, "It's cold."
Contrast G with:

H: This table is soluble in water.

If we never place the table in water, then H comes out true.
To be flammable is just, by definition, to have certain counterfactual properties.
These pajamas are flammable just in case they would burn if subjected to certain conditions.
Science demands counterfactual conditionals, so we can not merely ignore them.

IV. The problems of counterfactual conditionals bleed into indicative conditionals

(Note: This section requires more logical machinery than we have, right now.
But, it should make sense after September 17.)

Consider

I: If this is gold, then it is not water-soluble.
So, it is not the case that if this is gold then it is water-soluble.

Intuitively, this argument seems valid.
But, if we represent the argument as:

$G \supset {\sim}S \qquad / {\sim}(G \supset S)$

We get an invalid argument.
That is, the premises can be true while the conclusion is false.
The problem persists, even if I am referring to an actual fragment of gold.
So, even indicative conditionals expressing causal relations, as in science, suffer from problems of the material conditional.

V. The paradoxes of material implication

Consider:

J: If I am a man then some roses are red.

The material interpretation of J yields ‘true’.
This interpretation seems awkward because the antecedent lacks an appropriate connection with the consequent.
The natural-language “if...then...’ sometimes indicates causal connections.
Causal connections seem obviously missing in other examples, as well:

K: If I am a squirrel, then you are all chipmunks.

Even if the antecedent were true, we would have trouble determining the truth value of the whole.
Notice that the problem of the missing causal connection appears both in J, which is indicative, and K, which is counterfactual.
That is, the problems evinced by the counterfactual conditional are broader.
They are special instances of the so-called ‘paradoxes of material implication’.
The following three propositions are often called the paradoxes of material implication.

L: $P \supset (Q \supset P)$
M: $\sim P \supset (P \supset Q)$
N: $(P \supset Q) \vee (Q \supset P)$

All of these statements are laws of logic, which are also called tautologies, or logical truths.
Given the truth tables, logical truths are statements that can not possibly be false.

L says that if a statement is true, then anything implies it.
For, notice that the truth table with a true consequent is always true.
So, ‘if pigs fly, then Bush is still president’ is true, according to the material interpretation.
M says that if a statement is false, its negation entails anything.
We have already been looking at this problem, of the counterfactual conditional.
N says that for any statement, Q, either any other statement entails it, or it entails any statement.
For, Q can either be true or false.
If it is true, then, as in L, any statement entails it.
If it is false, then, as in M, it entails any statement.
So, ‘either it is raining implies Koalas eat bamboo or Koalas eat bamboo implies it is raining’ is a law of logic.

The paradoxes of material implication are uncomfortable.
Again, material implication does not represent causal connections.
Material implication represents logical connections.
So, it seems that ‘if...then...’ has two logical meanings.
The logical meaning of ‘⊃’ is truth-functional, encapsulated by the truth table.
But, given the results of §II, above, there does not seem to be a good alternative for the other meaning.
The causal meaning might not be truth-functional at all.

VI. Non-truth-functional operators

It simplifies our presentation of logic if we hold on to the truth-functional interpretation.
And there are good uses for material implication.

> O: If the alternate interior angles formed by two lines intersected by a third are congruent, then the two lines are parallel.
> P: If the hurricane hits, we will sustain great damage.

We could introduce a new operator, strict implication, ⇒, to take care of causal connections.
We could leave ‘P ⊃ Q’ as truth-functional, but take ‘P ⇒ Q’ as non-truth-functional.
So, consider again, sentence J.
We would take ‘M ⊃ R’ in the standard way.
Then, ‘M ⇒ R’ would lack a truth-value in the third and fourth rows.
C.I. Lewis proposed doing the semantics for strict implication modally, defining ‘P ⇒ Q’ as ‘□(P ⊃ Q)’.
‘(P · Q) ⊃ P’ and ‘(P · Q) ⇒ P’ both come out true, though.

Such a solution would leave many conditionals, including all counterfactual conditionals, without truth value in some situations.
We can leave the third and fourth of the truth table rows blank, neither true nor false.
Or, we can add a third truth value, often called undetermined, or indeterminate.
We will look at three-valued logics later in the term; it’s a good paper topic.

Another good topic would be to explore what are known as relevance logics.
In relevance logic, we insist that for a conditional to be true, its antecedent and consequent must be appropriately related.
This topic is well beyond our ken, at this point in the course.

Some other possible paper topics (see the course bibliography for references):

> Grice’s interpretation, which Fisher discusses
> C.I. Lewis’s operator
> Connections to three-valued logics
> Lewis Carroll’s paper, “A Logical Paradox”
> Goodman, and the relation between conditionals and scientific laws
> Frank Jackson and David Lewis have extended treatments of conditionals