Hamilton College
Russell Marcus
rmarcus1@hamilton.edu

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Translation Using Identity I (§8.7)

I. The identity predicate is a special predicate, with a special logic

Consider the following logical derivation:

1. Superman can fly.	Fs
2. Superman is Clark Kent.	???
So, Clark Kent can fly.	Fc

Identity, as in premise 2, is a relation among individuals.
We could write it 'Esc'.
But, identity has special logical properties, so we give it its own symbol, '='.
Identity sentences thus look a little different from other dyadic relations.

Clark Kent is Superman	c=s
Mary Ann Evans is George Eliot	m=g

But, they are just two-place relations.

To deny an identity, we can write either '~a=b' or 'a≠b'.
Negation applies to the identity predicate, and not to the objects related by that predicate.

We will discuss the special properties of the identity predicate on Monday.
Today, we will learn a bit of translating, using a group exercise called a jigsaw.

II. The jigsaw

Overview:
- Organize your base groups and divide tasks. (10 minutes)
- Go to work groups and learn something. (10 minutes)
- Go back to base groups and teach what you learned in the work groups to the other members of your base group. (25 minutes, 5 minutes per topic)

III. The worksheets

Identity Theory Jigsaw Lesson
Work Group: Only

I. Translation key:

a: Andy; d: Dwight; g: Angela; j: Jim; m: Michael; o: the Office; p: Pam; t: Toby
Ax: x is an accountant; Mx: x is a regional manager; Rx: x is a raise; Sx: x is a salesperson
Dxy: x despises y; Ixy: x is in y; Lxy: x loves y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Jim loves Pam.

   Ljp

2. Jim only loves Pam.

   Ljp • (x)(Ljx ⊃ x=p)

3. Only Andy and Dwight love Angela.

   Lag • Ldg • (x)[Lxg ⊃ (x=a ∨ x=d)]

4. There is only one accountant in the office.

   (∃x){Ax • Ixo • (y)[(Ay • Iyo) ⊃ y=x)]}

5. Only Michael would give Angela a raise.

   (∃x)(Rx • Gmxa) • (x)[Rx ⊃ (y)(Gyxa ⊃ y=m)]

III. Try these, using the key in I.

6. Michael is the only regional manager.

7. There is only one salesperson who despises Toby.

8. Only Dwight and Jim are salespeople in the office.

Identity Theory Jigsaw Lesson
Work Group: Except

I. Translation key:

c: Creed; g: Angela; m: Michael; n: Jan; p: Pam; o: the Office; r: Scranton; s: Stanley; t: Toby
Ax: x is an accountant; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Px: x is a person; Sx: x is a salesperson; Tx: x is a product
Ixy: x is in y; Kxy: x likes y; Lxy: x loves y; Pxy: x passed y; Sxy: x sells y; Txy: x tolerates y; Vxy: x lives in y
Gxyz: x would give y to z

II. Examine the translations below, which use the key in I.

1. Everyone loves Pam.

   $(x)(Px \supset Lxp)$

2. Everyone except Angela loves Pam.

   $Pa \bullet {\sim}Lap \bullet (x)[(Px \bullet x{\neq}a) \supset Lxp]$

3. Someone likes all employees except Toby.

   $Et \bullet (\exists x)\{Px \bullet {\sim}Kxt \bullet (y)[(Ey \bullet y{\neq}t) \supset Kxy]\}$

4. Everyone in the office except Pam lives in Scranton.

   $Pp \bullet Ipo \bullet {\sim}Vps \bullet (x)[(Px \bullet Ixo \bullet x{\neq}p) \supset Vxs]$

5. Everyone but Creed passed a drug test.

   $Pc \bullet (x)(Dx \supset {\sim}Pcx) \bullet (x)[(Px \bullet x{\neq}c) \supset (\exists y)(Dy \bullet Pxy)]$

III. Try these, using the key in I.

6. All employees are happy except Stanley.

7. No one except Michael tolerates Jan.

8. Some products are sold by all employees except Michael.

Identity Theory Jigsaw Lesson
Work Group: Superlatives

I. Translation key:

c: Creed; d: Dwight; j: Jim; m: Michael; n: Jan; p: Pam; r: the Scranton branch; u: the Utica branch
Ax: x is an accountant; Bx: x is a branch; Ex: x is an employee; Ox: x is an office; Sx: x is a salesperson
Bxy: x is bigger than y; Hxy: x has y; Ixy: x is in y; Mxy: x is smaller than y; Nxy: x is nicer than y; Zxy: x is lazier than y
Nxyz: x is nearer than y to z.

II. Examine the translations below, which use the key in I.

1. Jim is a nicer salesperson than Dwight.

 Sj • Sd • Njd

2. Jim is the nicest salesperson.

 Sj • (x)[(Sx • x≠j) ⊃ Njx]

3. Utica is the smallest branch.

 Bu • (x)[(Bx • x≠u) ⊃ Mux]

4. Creed is the laziest employee in the office.

 Ec • Ico • (x)[(Ex • Ixo • x≠c) ⊃ Zcx]

5. Michael is the employee who has the biggest office.

 Em • (∃x){(Ox • Hmx) • (y){(Ey • y≠m) ⊃ (z)[(Oz • Hyz) ⊃ Bxz)]}}

III. Try these, using the key in I.

6. Scranton is the biggest branch.

7. Utica is the nearest branch to the Scranton branch.

8. Some employee is the biggest accountant in the office.

Identity Theory Jigsaw Lesson
Work Group: At Least

I. Translation key:

j: Jim; o: the Office
Ax: x is an accountant; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Ix: x is in the office
Bxy: x is bigger than y; Ixy: x is in y; Pxy: x passed y; Txy: x tolerates y

II. Examine the translations below, which use the key in I.

1. There is at least one accountant in the office.

$(\exists x)(Ax \bullet Ixo)$

2. There are at least two accountants in the office.

$(\exists x)(\exists y)(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet x{\neq}y)$

3. There are at least three accountants in the office.

$(\exists x)(\exists y)(\exists z)(Ax \bullet Ixo \bullet Ay \bullet Iyo \bullet Az \bullet Izo \bullet x{\neq}y \bullet x{\neq}z \bullet y{\neq}z)$

4. There are at least two happy employees who tolerate each other.

$(\exists x)(\exists y)(Hx \bullet Ex \bullet Hy \bullet Ey \bullet x{\neq}y \bullet Txy \bullet Tyx)$

5. At least three accountants passed a drug test.

$(\exists x)(\exists y)(\exists z)[Ax \bullet Ay \bullet Az \bullet x{\neq}y \bullet x{\neq}z \neq y{\neq}z \bullet (\exists w)(Dw \bullet Pxw) \bullet (\exists w)(Dw \bullet Pyw) \bullet (\exists w)(Dw \bullet Pzw)]$

III. Try these, using the key in I.

6. There are at least two employees bigger than Jim.

7. There are at least three employees bigger than Jim.

8. There are at least four accountants in the office.

Identity Theory Jigsaw Lesson
Work Group: At Most

I. Translation key:

a: Andy; d: Dwight; g: Angela; m: Michael; o: the Office
Ax: x is an accountant; Ex: x is an employee; Mx: x is a regional manager; Px: x is a person
Axy: x is y's assistant; Bxy: x is bigger than y; Hxy: x has y; Ixy: x is in y; Kxy: x likes y

Note: 'At most' statements make no existential commitments.

II. Examine the translations below, which use the key in I.

1. At most one person is Michael's assistant.

   (x)(y)[(Px • Axm • Py • Aym) ⊃ x=y]

2. At most two employees are accountants.

   (x)(y)(z)[(Ex • Ax • Ey • Ay • Ez • Az) ⊃ (x=y ∨ x=z ∨ y=z)]

3. At most two people are Michael's assistants.

   (x)(y)(z)[(Px • Axm • Py • Aym • Pz • Azm) ⊃ (x=y ∨ x=z ∨ y=z)]

4. There is at most one accountant in the office bigger than Dwight.

   (x)(y)[(Ax • Ixo • Bxd • Ay • Iyo • Byd) ⊃ x=y]

5. At most two regional managers have employees bigger than Andy.

   (x)(y)(z){[Mx • (∃w)(Ew • Hxw • Bwa) • My • (∃w)(Ew • Hyw • Bwa) • Mz • (∃w)(Ew • Hzw • Bwa)] ⊃ (x=y ∨ x=z ∨ y=z)}

III. Try these, using the key in I.

6. There is at most one accountant in the office.

7. There are at most three accountants in the office.

8. Some people like Angela, but at most two.

VI. Solutions

Answers to the 'Try these' examples on each worksheet

Translation key for all problems on all five worksheets:

a: Andy; c: Creed; d: Dwight; g: Angela; j: Jim; m: Michael; n: Jan; o: the Office; p: Pam; r: the Scranton branch; s: Stanley; t: Toby; u: the Utica branch
Ax: x is an accountant; Bx: x is a branch; Dx: x is a drug test; Ex: x is an employee; Hx: x is happy; Mx: x is a regional manager; Ox: x is an office; Px: x is a person; Rx: x is a raise; Sx: x is a salesperson; Tx: x is a product
Axy: x is y's assistant; Bxy: x is bigger than y; Dxy: x despises y; Fxy: x farms y; Hxy: x has y; Ixy: x is in y; Kxy: x likes y; Lxy: x loves y; Mxy: x is smaller than y; Nxy: x is nicer than y; Pxy: x passed y; Sxy: x sells y; Txy: x tolerates y; Vxy: x lives in y; Zxy: x is lazier than y
Gxyz: x would give y to z; Nxyz: x is nearer than y to z.

Only
6. Mm • (x)(Mx ⊃ x=m)
7. (∃x){Sx • Dxt • (y)[(Sy • Dyt) ⊃ y=x]}
8. Sd • Ido • Sj • Ijo • (x)[(Sx • Ixo) ⊃ (x=d ∨ x=j)]

Except
6. Es • ~Hs • (x)[(Ex • x≠s) ⊃ Hs]
7. Pm • Tmn • (x)[(Px • x≠m) ⊃ ~Txn]
8. Em • (∃x){Tx • ~Smx • (y)[(Ey • y≠m) ⊃ Syx]}

Superlatives
6. Br • (x)[(Bx • x≠r) ⊃ Brx]
7. Br • Bu • (x)[(Bx • x≠u) ⊃ Nuxs]
8. (∃x){Ex • Ixo • Ax • (y)[(Ay • Iyo • y≠x) ⊃ Bxy]}

At least
6. (∃x)(∃y)(Ex • Ey • x≠y • Bxj • Byj)
7. (∃x)(∃y)(∃z)(Ex • Ey • Ez • Bxj • Byj • Bzj • x≠y • x≠z • y≠z)
8. (∃x)(∃y)(∃z)(∃w)(Ax • Ixo • Ay • Iyo • Az • Izo • Aw • Iwo • x≠y • x≠z • x≠w • y≠z • y≠w • z≠w)

At most
6. (x)(y)[(Ax • Ixo • Ay • Iyo) ⊃ x=y]
7. (x)(y)(z)(w)[(Ax • Ixo • Ay • Iyo • Az • Izo • Aw • Iwo) ⊃ (x=y ∨ x=z ∨ x=w ∨ y=z ∨ y=w ∨ z=w)]
8. (∃x)(Px • Kxa) • (x)(y)(z)[(Px • Kxa • Py • Kya • Pz • Kza) ⊃ (x=y ∨ x=z ∨ y=z)]