Philosophy 240: Symbolic Logic
Fall 2009
Mondays, Wednesdays, Fridays: 9am - 9:50am

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Class 31 - November 9
Translation Using Relational Predicates I (§8.6)

I. Introducing Relational Predicates

Consider:

> Bob is taller than Charles.
> Andrew is taller than Bob.
> For any x, y and z, if x is taller than y and y is taller than z, then x is taller than z.
> So, Andrew is taller than Charles.

The conclusion should follow logically, but how do we translate the predicates?
If we only have monadic (1-place) predicates, like the ones we have so far considered, we have to translate the two first sentences with two different predicates:

> Bob is taller than Charles: Tb
> Andrew is taller than Bob: Ya

We really want a predicate that takes two objects. This is called a dyadic predicate. For examples:

> Txy: x is taller than y
> Kxy: x knows y
> Bxy: x believes y
> Dxy: x does y

We can have three-place predicates too, called triadic predicates:

> Gxyz: x gives y to z
> Kxyz: x kisses y in z
> Bxyz: x is between y and z

We can construct four-place and higher-place predicates.
All predicates which take more than one object are called relational, or polyadic.

By introducing relational predicates, we have extended our language.
We are now using a language I called **F**, for Full First-Order Predicate Logic, rather than **M**.
For the purposes of this course, the differences between **F** and **M** are minor.
The only significant difference in the formation rules is in the construction of atomic formulas, at step 1.
But, there are significant differences in the properties of the proof theories we will introduce in those languages.
Call a theory decidable when one can determine, by an effective (or algorithmic) method whether it is a theorem.
The theory based on **M** is decidable, while the theory based on **F** is undecidable.

Formation rules for wffs of F

1. An n-place predicate followed by n terms is a wff.
2. If α is a wff, so are
   (∃x)α, (∃y)α, (∃z)α, (∃w)α, (∃v)α
   (x)α, (y)α, (z)α, (w)α, (v)α
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
5. These are the only ways to make wffs.

Remember that terms, for now, are either constants or variables; later we will add functions.
Note that you can determine the value of 'n' in an n-place predicate precisely by counting the number of terms that follow the predicate letter.

II. Exercises A. Translate each sentence into predicate logic, using relational predicates.

1. John loves Mary
2. Tokyo isn't smaller than New York.
3. Marco was introduced to Erika by Paco.
4. America took California from Mexico.

III. Quantifiers with relational predicates

Consider again the original argument.
We can now translate the first two premises and the conlcusion.

Bob is taller than Charles: Tbc
Andrew is taller than Bob: Tab
Andrew is taller than Charles: Tac

But we need quantifiers for the relations in the third premise.

Let's start with some sentences with just one quantifier.
The following four sentences use 'Bxy' for 'x is bigger than y'.

Joe is bigger than some thing : (∃x)Bjx
Something is bigger than Joe: (∃x)Bxj
Joe is bigger than everything: (x)Bjx
Everything is bigger than Joe: (x)Bxj

Next, we can introduce overlapping quantifiers.
Consider: 'Everything loves something', using 'Lxy' for 'x loves y': (x)(∃y)Lxy
Note the different quantifier letters: overlapping quantifiers must use different variables.
Also, the order of quantifiers matters
'(∃x)(y)Lxy' means that something loves everything, which is different.
Returning to the original argument, we now have:

1. Tbc
2. Tab
3. (x)(y)(z)[(Txy • Tyz) ⊃ Txz] / Tac

After we finish translations, we will return to deriving the conclusion of this argument.
Here are some more examples:

1. Something taught Plato. (Txy: x taught y)
   (∃x)Txp
2. Someone taught Plato. (Px: x is a person)
   (∃x)(Px · Txp)
3. Plato taught everyone.
   (x)(Px ⊃ Tpx)
4. Everyone knows something. (Kxy: x knows y)
   (x)[Px ⊃ (∃y)Kxy]
5. Jen reads all books written by Asimov. (Bx: x is a book; Wxy: x writes y; Rxy: x reads y; j: Jen; a: Asimov)
   (x)[(Bx • Wax) ⊃ Rjx]
6. Some people read all books written by Asimov.
   (∃x){Px • (y)[(By • Way) ⊃ Rxy]}
7. Some people read all books written by some one.
   (∃x){Px • (∃y){Py • (z)[(Bz • Wyz) ⊃ Rxz]}}
8. Honest candidates are always defeated by dishonest candidates. (Hx, Cx, Dxy: x defeats y)
   (x){(Cx · Hx) ⊃ (∃y)[(Cy · ~Hy) · Dyx]}
9. No mouse is mightier than himself. (Mx, Mxy: x is mightier than y)
   (x)(Mx ⊃ ~Mxx)
10. Everyone buys something from some store. (Px, Sx, Bxyz: x buys y from z)
    (x)[Px ⊃ (∃y)(∃z)(Sz · Bxyz)]
11. No store has everyone for a customer.
    ~(∃x){Sx · (y)[Py ⊃ (∃z)Byzx]} or (x){Sx ⊃ (∃y)[Py · (z)~Byzx]}

IV. Exercises B. Translate each of the following into predicate logic, using relational predicates.

1. Everyone is wiser than someone. (Wxy: x is wiser than y)
2. Someone is wiser than everyone.
3. Some financier is richer than everyone. (Fx, Rxy: x is richer than y)
4. No deity is weaker than some human. (Dx, Hx, Wxy: x is weaker than y)
5. There is a store from which everyone buys something. (Px, Sx, Bxyz: x buys y from z)

V. Solutions

Answers to Exercises A:
1. Ljm
2. ~Stn
3. Ipme
4. Tcam

Anwers to Exercises B:
1. (x)[Px ⊃ (∃y)(Py · Wxy)]
2. (∃x)[Px · (y)(Py ⊃ Wxy)]
3. (∃x)[Fx · (y)(Py ⊃ Rxy)]
4. ~(∃x)[Dx · (∃y)(Hy · Wxy)] or (x)[Dx ⊃ (y)(Hy ⊃ ~Wxy)]
5. (∃x){Sx · (y)[Py ⊃ (∃z)Byzx]}