Philosophy 240: Symbolic Logic
Fall 2009
Mondays, Wednesdays, Fridays: 9am - 9:50am

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Class 24 - October 23
Philosophy Friday #5:Truth and Liars
Fisher, pp 36-39; 125-131

I. The liar

In Philosophy Friday #3: Three-Valued Logics, we discussed eight motivations for three-valued logics.
The fourth motivation for introducing a third truth-value was to avoid semantic paradoxes.
The most important semantic paradox is called the liar.

1. This sentence is false.

1 is an example of a paradoxical sentence.
If 1 is true, then it is false, which makes it true, which makes it false...
1 seems to lack a definite truth value, even though it is a perfectly well-formed sentence.
It is often called Epimenides' paradox.
Epimenides was a Cretan to whom the statement that all Cretans are liars is attributed.

Quine, in his essay "The Ways of Paradox," argues that there are grounds to question either the paradoxicality or the well-formedness of 1.
It is not clear what 'This sentence' refers to.
If we substitute 'This sentence is false' for 'This sentence', then we get

2. 'This sentence is false' is false.

Now, 2 does not ascribe falsity to itself, and the paradox is gone.
In response, Quine constructed 3, which avoids the above problem.

3. 'Yields falsehood when appended to its own quotation' yields falsehood when appended to its own quotation.

In both 1 and 3, the culprit seems to be falsity.
Truth and falsity are called semantic terms; 'semantic' can refer to truth or to meaning.
In ordinary usage, when we talk about semantics, we refer to meanings.
When we present a semantics for a formal language, we provide truth conditions for the wffs of the language.
Our semantics for **PL**, for example, consisted of truth tables.
Soon we will look a bit at the more complicated semantics for **M**.

One diagnosis of the liar is that it involves illicit self-reference.
Another self-referential paradox, the barber paradox, is due to Bertrand Russell, though he credits an anonymous source.
Consider the barber in a town who shaves all the men who do not shave themselves.
Does he shave himself?
You can construct a puzzling declarative sentence, which I leave to you as an exercise.

II. Grelling's paradox

Not all semantic paradoxes involve truth, or self-reference.
Grelling's paradox is semantic, but does not involve 'truth' or 'falsity' explicitly.
Grelling's paradox is about meaning.
Note that some predicates apply to themselves, whereas others do not.
'Polysyllabic' is polysyllabic; 'monosyllabic' is not monosyllabic.
Call a predicate heterological if it does not apply to itself.
'Monosyllabic' is heterological; 'polysyllabic' is not heterological.
(We can call it autological, or homological.)
Now, consider whether 'heterological' applies to itself.
If it does, then 'heterological' is not heterological.
But, if 'heterological' is not heterological, then it does not apply to itself, which means that it is heterological.
Here we go again!
We can construct a statement involving 'heterological' whose truth value is puzzling.

4. 'Heterological' is heterological.

There are two popular solutions to semantic paradoxes

5. Introduce a third truth value for paradoxical sentences.
6. Banish semantic terms from formal languages.

There are two problems with introducing a third truth value.
First, as we saw, our formal system either loses logical truths and valid inferences (on K_3) or it ascribes truth to conditional sentences with indeterminate antecedents and consequences (on L_3).
Second, adding a third truth value will not solve the problem of the strengthened liar.

7. This sentence is not true.

See the paper topics below for references to the strengthened liar, or you can google it.
We will look a bit at the second popular solution, which derives from Tarski's work on truth.
First, we should sketch the basic theories of truth; see Fisher for a slightly more extended discussion.

III. Truth

The standard concept of truth is called the correspondence theory.
The correspondence theory of truth traces back at least to Plato, though it is traditional to ascribe it to Aristotle.

> To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true (*Metaphysics*, 1011b25).

According to the correspondence theory of truth, truth is a relation between words and worlds.
The truth of a sentence consists in its agreement with (or correspondence to) reality.

Fisher discusses the traditional rival to correspondence theories, coherence theories of truth.
According to a coherentist, the truth of a sentence consists in its consistency with other beliefs that we hold.
Since different people hold different beliefs, the coherence truth of a sentence depends on the person who is considering the sentence.
Imagine that I believe in a traditional, monotheistic God and you do not.
8 will be true for me, since it coheres with my other beliefs.

8. God is omniscient.

In contrast, 8 will be false for you, since it conflicts with your other beliefs.
Coherence theories thus collapse into relativism, and I will not consider them any further.

The correspondence theory is an inflationary theory of truth.
The inflationary theory is distinguished from a third theory that Fisher discusses, the deflationary theory of truth.
Deflationary theories of truth were developed in the last century, at least under that name, or under the name 'minimalism'.
The deflationary theory of truth has many proponents, all of whom have different ways of understanding and explaining deflationism.
Deflationists believe that there is no essence to truth, no single reduction of truth to a specific property.

Some deflationists claim that truth is just a device for simplifying long conjunctions.
If you said a lot of smart things at the party, I could list them all.
Or, I could just say:

9. Everything you said last night was true.

In 9, 'true' is eliminable, by a longer, clunky sentence, or set of sentences.
Such eliminations are essential to the deflationary conception of truth.
Sometimes, the deflationary theory is called a redundancy theory: to say that snow is white is true is just to say, redundantly, that snow is white.
So, according to the deflationist, we do not need 'true'; it just comes in handy.

IV. Explosion

In the early twentieth century, truth had gotten a terrible reputation, in large part due to the paradoxes.
The paradoxes lead to contradictions.
In traditional, or classical, formal systems, as we have seen, contradictions entail everything.
In class, I called this property of classical systems explosion.

10.	1. P • ~P	/ Q
	2. P	1, Simp
	3. P ∨ Q	2, Add
	4. ~P	1, Com, Simp
	5. Q	3, 4, DS

QED

The excitement surrounding the new logic of the early twentieth century included hopes that all human knowledge could be represented by formal languages.
Since contradictions lead to explosion and formal languages in which the paradoxes are representable lead to contradictions, it became seen as essential to avoid formalizing the notion of truth.
Since formal languages were seen as the locus of all of our knwoledge, it seemed that truth was just not a legitimate term.
The bad reputation of truth explains the interest of many philosophers in the relativism of coherence truth.
All recent work on truth, whether deflationary or inflationary, owes its origins to Tarski, who, in the 1930s, showed how to rehabilitate the concept of truth within formalized languages, how to avoid explosion without giving up on a formalized notion of truth.

V. The T-schema

Both inflationists and deflationists agree that a minimal condition for truth is what we call the T-schema, or Convention T, following Tarski.

10. p is true iff x

In 10, 'p' is the name of any sentence, and x are the truth conditions of that sentence.

We can use the T-schema to describe the truth conditions for any sentence.
Here are some instances of the T-schema:

11. 'the cat is on the mat' is true iff the cat is on the mat.
12. '2+2=4' is true iff 2+2=4
13. 'Barack Obama is president' is true iff the husband of Michelle Obama and father of Sasha Obama and Malia Obama is head of the executive branch of the United States of America.

Note that, as in 13, the truth conditions do not have to be expressed using the same terms as the sentence on the left.
We can even use a different language for the sentence and for its truth conditions.
Consider:

14. 'El gato está en el alfombrilla' is true iff the cat is on the mat.

Notice that you could understand the truth conditions of 14 without understanding the meaning of the Spanish sentence on the left side.

VI. The return of the liar

To see how the liar leads to a contradiction, consider 1 again.
Applying the T-schema yields:

15. 1 is true iff 1 is false.

We can translate this sentence into **M** by taking a constant, say 'p', to stand for the sentence 1, and introducing a truth predicate, 'T'.
We also have to take 'P is true' to be the negation of 'P is false'; the strengthened liar will work a bit differently.

So, we get:

16.	1. Tp ≡ ~Tp	From the T-schema and the definition of P
	2. (Tp ⊃ ~Tp) • (~Tp ⊃ Tp)	1, Equiv
	3. ~Tp ⊃ Tp	2, Com, Simp
	4. Tp ∨ Tp	3, Impl, DN
	5. Tp	4, Taut
	6. Tp ⊃ ~Tp	2, Simp
	7. ~Tp ∨ ~Tp	6, Impl
	8. ~Tp	7, Taut
	9. Tp • ~Tp	5, 8, Conj

Tilt!

VII. Tarski's solution

Tarski's solution to the liar paradox is to distinguish between an object language and a meta-language, and to rule sentence 1 out of the object language.
The object language is the language in which we are working.
The meta-language is a language in which we can talk about the object language.
Instances of the T-schema are sentences of the meta-language which we can use to characterize truth within the object language.
Since we are constructing a formal language, like **PL** or **M**, we can choose to include or exclude any terms.

Deleting just the liar from the object language, though, appears ad hoc.
Tarski claims that all uses of the term 'true' are inadmissible from the object language.
A language can not contain its own truth predicate.
To determine which sentences of an object language are true and which are false, we have to examine the truth conditions as given on the right hand side of instances of the T-schema.
The truth conditions are written in the metalanguage.

The key to Tarski's solution to the liar paradox is that sentences like 1 are ill-formed, since they include 'false' in the object language.
When I want to use a sentence like 9, I ascend to a metalanguage to do so.
In a metalanguage, I can also construct sentences like the important 17.

17. All consequences of true sentences are true.

Sentences like 17 are fundamental to metalogic, and model theory, fields that Tarski basically created.
Metalogic is mainly studied in advanced logic classes, though we saw some metalogic in Philosophy Friday #4: Adequacy.
In metalogic, we explore questions of whether a formal system is sound, or complete, or decidable.

We will put them aside, here, and see if Tarski's T-schema can help us understand or ordinary conception of truth.

VIII. Is truth deflationary or inflationary?

Tarski uses sentences like 9 and 17 to show that 'truth' plays an essential role in a theory.
It might thus seem like Tarski is an inflationist, a correspondence theorist.
But Tarski's claim that 'truth' is essential may not have inflationary implications.
If 'true' is a device used to refer to other sentences, it depends on what we think of those other sentences, the ones without 'true' and with content.
If we need a words-worlds relation in order to ascribe 'true' to a sentence, then truth will not be merely deflationary, or redundant.
If all there is to truth is eliminable, then perhaps there is no essence to truth.
Aristotle's original claim could be given a deflationary interpretation itself!

Tarski prescribes the following method to determining the correct notion of truth, in "The Semantic Conception of Truth: and the Foundations of Semantics," a copy of which can be found on the website.

> It seems to me obvious that the only rational approach to [questions about the correct notion of truth] would be the following: We should reconcile ourselves with the fact that we are confronted, not with one concept, but with several different concepts which are denoted by one word; we should try to make these concepts as clear as possible (by means of definition, or of an axiomatic procedure, or in some other way); to avoid further confusions, we should agree to use different terms for different concepts; and then we may proceed to a quiet and systematic study of all concepts involved, which will exhibit their main properties and mutual relations (p 355).

Furthermore, Tarski believes that the semantic conception is agnostic among any deeper philosophical debates.

> We may accept the semantic conception of truth without giving up any epistemological attitude we may have had; we may remain naive realists, critical realists or idealists, empiricists or metaphysicians - whatever we were before. The semantic conception is completely neutral toward all these issues (p 362).

Deflationists look at the T-schema as a satisfactory definition of truth.
That's why deflationism also goes by the name 'redundancy theory'.
Inflationists about truth look at the T-schema as merely a minimal condition for truth.
They claim that there are additional requirements, like correspondence to reality.
Hartry Field's paper "Tarski's Theory of Truth" establishes convincingly that Tarski is not a deflationist; it's a difficult and technical paper.

Much of the contemporary work on truth and the paradoxes is similarly technical, though the classical discussion of theories of truth is mainly philosophical.
One of the more controversial but productive areas of recent research has been dialetheism.
According to dialetheists like Graham Priest, the liar is both true and false.
There has been a lot of technical work on paraconsistent logics, logics which contain contradictions.
Contradictions in classical logic are explosive: anything follows.

So, dialetheists look to block explosion.
A paper on dialetheism could be interesting.

IX. A few paper ideas

1. Does introducing a third truth-value solve the problem of the liar? Discuss the strengthened liar paradox. Kirkham has a good, if brief, discussion of the strengthened liar.

2. Is truth deflationary or inflationary? Fisher discusses this topic, and provides some good references. See Horwich, and the Lynch collection.

3. Is truth relative to a language? Tarski's definition of truth introduces a different truth predicate for each language, and creates a hierarchy of languages. Is this construction objectionable? See Fisher, Kirkham, and the Lynch collection.

4. Graham Priest has lately been defending dialetheism, the claim that there can be true contradictions. Can there be true contradictions?

X. Readings

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Priest, Graham, JC Beall and Bradley Armour-Garb. *The Law of Non-Contradiction: New Philosophical Essays*. Oxford: Clarendon Press, 2004. See especially the introduction, and Essays 1, 7, and 12, among others.
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