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Derivations in Predicate Logic I (§8.2)

There are four rules for deductions in predicate logic, in addition to the ones used in propositional logic.

I. Taking off the universal quantifier

Recall the argument:

All philosophers are happy:	(x)(Px ⊃ Hx)
Emily is a philosopher:	Pe
So, Emily is happy:	He

We need to add a rule that will generate this conclusion out of these premises.

Rule #1: Universal Instantiation (UI)

Remove the leading universal quantifier.
Replace all occurrences of variables bound by that quantifier with either a variable (x, y, z) or a constant (a, b, c).
Use only when the main connective is a quantifier, i.e. on whole lines.
We write this rule as:

(x)$\mathscr{F}$x / $\mathscr{F}$y
(x)$\mathscr{F}$x / $\mathscr{F}$a

Binding Rule: When you use instantiate or generalize, you must change all the bound variables in the same way.

So '(x)[Sx ∨ (Pa · Tx)]' can be instantiated as:
Sa ∨ (Pa · Ta)
Sb ∨ (Pa · Tb)
Sx ∨ (Pa · Tx)
Sy ∨ (Pa · Ty)

But it can not be instantiated as:
Sa ∨ (Pa · Tb)
Sx ∨ (Pa · Ta)

Returning to the argument above:

1. (x)(Px ⊃ Hx)		
2. Pe	/He	
3. Pe ⊃ He		1, UI
4. He		3, 2, MP

QED

II. Putting on the universal quantifier

Consider:

1. Everything happy is content
2. No miser is content.

∴ No miser is happy.

Regiment this as:

1. (x)(Hx ⊃ Cx)
2. (x)(Mx ⊃ ~Cx)   /(x)(Mx ⊃ ~Hx)

We need to remove the quantifiers and put them back on
What if we introduce a rule such as: '$\mathscr{F}$a / (x)$\mathscr{F}$x' ?
Consider 'Pa/ (x)Px' where we interpret 'P' to mean 'is portly' and 'a' to mean 'Adam'.
We're concluding that everything is portly from just one instance.
This is called the Fallacy of Hasty Generalization.
Solution: Never universally quantify over a constant.
That is, you may not replace a constant with a variable bound by a universal quantifier.
That keeps us from ever quantifying over too few cases.
But quantifying over a variable is acceptable.
You're not committing the same fallacy, because the variable can stand for anything and everything.

Rule #2: Universal Generalization (UG)
Place a universal quantifier in front of a whole statement, so that the scope of the quantifier is now the whole statement.
Replace all occurrences of the variable over which you are quantifying with the variable in the quantifier.
That is, bind all instances of the variable.
Warning: you must replace all occurrences!

	$\mathscr{F}$y	/ (x)$\mathscr{F}$x
Not	$\mathscr{F}$a	/ (x)$\mathscr{F}$x

For example:

1. (x)(Hx ⊃ Cx)		
2. (x)(Mx ⊃ ~Cx)	/ (x)(Mx ⊃ ~Hx)	
3. Hy ⊃ Cy		1, UI
4. My ⊃ ~Cy		2, UI
5. ~Cy ⊃ ~Hy		3, Trans
6. My ⊃ ~Hy		4, 5, HS
7. (x)(Mx ⊃ ~Hx)		6, UG

QED

III. Putting on the existential quantifier

Consider: Oscar is a Costa Rican. So there are Costa Ricans.

Rule #3: Existential Generalization (EG)
Place an existential quantifier in front of any statement.
Change any or all occurrences of the variable over which you are quantifying with the quantifier letter.

$\mathscr{F}$a/ (∃x)$\mathscr{F}$x
$\mathscr{F}$y/ (∃x)$\mathscr{F}$x

So:

1. Co / (∃x)Cx
2. (∃x)Cx 1, EG

QED

IV. Taking off the existential quantifier

Consider: All New Yorkers are Americans. Some New Yorkers are bald. So, some Americans are bald.

(x)(Nx ⊃ Ax)
(∃x)(Nx · Bx) /(∃x)(Ax · Bx)

We have to take off the '∃x' in the second premise.
The '∃x' only commits us to the existence of one thing.
So, when we take it off, we have to put on a constant.
We can't have said anything about that constant earlier - it has to be a new thing!

Rule #4: Existential Instantiation (EI)
Remove the leading existential quantifier.
Replace all occurrences which were bound by the quantifier with the same, new constant.

	(∃x)$\mathscr{F}$x	/ $\mathscr{F}$a, where a is a new name
Not:	(∃x)$\mathscr{F}$x	/ $\mathscr{F}$y

About new names, consider what would happen without that restriction:

1. (∃x)(Fx · Px)		(Some fruits are pears)
2. (∃x)(Fx · Ox)		(Some fruits are oranges)
3. Fa · Pa	1, EI	
4. Fa · Oa	2, EI: but wrong!	
5. Pa	3, Com, Simp	
6. Oa	4, Com, Simp	
7. Pa · Oa	5, 6, Conj	
8. (∃x)(Px · Ox)	7, EG	(Some pears are oranges. Uh-oh!)

Since we have this restriction on EI, but not on UI, always EI before you UI!

A sample derivation:

1. (x)(Nx ⊃ Ax)	
2. (∃x)(Nx · Bx)	/ (∃x)(Ax · Bx)
3. Na · Ba	2, EI
4. Na ⊃ Aa	1, UI
5. Na	3, Simp
6. Aa	4, 5, MP
7. Ba	3, Com, Simp
8. Aa · Ba	6, 7, Conj
9. (∃x)(Ax · Bx)	8, EG

QED

V. **Exercises A**. Derive the conclusions of each of the following arguments:

1.
1. (x)(Ax ⊃ Bx)
2. (∃x)(Ax) /(∃x)(Bx)

2.
1. (∃x)(~Fx · ~Gx)
2. (x)(~Gx ⊃ ~Hx) /(∃x)~(Fx ∨ Hx)

3.
1. (x)(Jx ∨ ~Kx)
2. (x)~Jx /(x)~Kx

Solutions will vary.