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Rules of Implication II (§7.2)

I. Four more Rules of Implication

These four forms are valid. Check them using the indirect truth table method.

1. *Conjunction* (Conj)

α
$\beta \qquad / \alpha \cdot \beta$

For example, "I have peas. I have carrots. So I have peas and I have carrots."

2. *Addition* (Add)

$\alpha \qquad / \alpha \vee \beta$

For example, "I have peas. So, either I have peas or Barney is a purple rabbit."

3. *Simplification* (Simp)

$\alpha \cdot \beta \qquad / \alpha$

For example, "I have peas and I have carrots. So I have peas."

4. *Constructive Dilemma* (CD)

$(\alpha \supset \beta) \cdot (\gamma \supset \delta)$
$\alpha \vee \gamma \qquad / \beta \vee \delta$

Note the similarity between CD and Modus Ponens.
From the conjunction of two conditionals, and the disjunction of their antecedents, one can infer the disjunction of their consequents.

Be careful to avoid these two *invalid* inferences!

$\alpha \qquad / \alpha \cdot \beta$

$\alpha \vee \beta \qquad / \alpha$

II. Truth Table for Constructive Dilemma

I have mentioned that you can check each of the rules of implication for validity by using the indirect truth table method.
Here is a full truth table for CD.
You can see that there is no line on which the conclusion is false and the premises are both true.

(α	$\supset$	β)	$\bullet$	(γ	$\supset$	δ)	/	α	$\vee$	γ	//	β	$\vee$	δ
T	T	T	**T**	T	T	T		T	**T**	T		T	**T**	T
T	T	T	**F**	T	F	F		T	**T**	T		T	**T**	F
T	T	T	**T**	F	T	T		T	**T**	F		T	**T**	T
T	T	T	**T**	F	T	F		T	**T**	F		T	**T**	F
T	F	F	**F**	T	T	T		T	**T**	T		F	**T**	T
T	F	F	**F**	T	F	F		T	**T**	T		F	**F**	F
T	F	F	**F**	F	T	T		T	**T**	F		F	**T**	T
T	F	F	**F**	F	T	F		T	**T**	F		F	**F**	F
F	T	T	**T**	T	T	T		F	**T**	T		T	**T**	T
F	T	T	**F**	T	F	F		F	**T**	T		T	**T**	F
F	T	T	**T**	F	T	T		F	**F**	F		T	**T**	T
F	T	T	**T**	F	T	F		F	**F**	F		T	**T**	F
F	T	F	**T**	T	T	T		F	**T**	T		F	**T**	T
F	T	F	**F**	T	F	F		F	**T**	T		F	**F**	F
F	T	F	**T**	F	T	T		F	**F**	F		F	**T**	T
F	T	F	**T**	F	T	F		F	**F**	F		F	**F**	F

III. Examples of derivations using the new rules of implication

A derivation using Conj and Simp:

1. $A \supset B$	
2. $F \supset D$	
3. $A \cdot E$	
4. $\sim D$	/ $B \cdot \sim F$
5. A	3, Simp
6. B	1, 5, MP
7. $\sim F$	2, 4, MT
8. $B \cdot \sim F$	6, 7, Conj

QED

A derivation using Add:

1. $\sim M \vee N$	
2. $\sim\sim M$	/ $N \vee O$
3. N	1, 2, DS
4. $N \vee O$	3, Add

QED

A derivation using CD

1. $N \supset (O \cdot P)$		
2. $(Q \cdot R) \supset O$		
3. $N \vee (Q \cdot R)$	/ $(O \cdot P) \vee O$	
4. $[N \supset (O \cdot P)] \cdot [(Q \cdot R) \supset O]$		1, 2, Conj
5. $(O \cdot P) \vee O$		4, 3, CD

QED

A longer derivation:

1. $(\sim A \vee B) \supset (G \supset D)$	
2. $(G \vee E) \supset (\sim A \supset F)$	
3. $A \vee G$	
4. $\sim A$	/ $F \cdot D$
5. G	3, 4, DS
6. $G \vee E$	5, Add
7. $\sim A \supset F$	2, 6, MP
8. F	7, 4, MP
9. $\sim A \vee B$	4, Add
10. $G \supset D$	1, 9, MP
11. D	10, 5, MP
12. $F \cdot D$	8, 11, Conj

QED

IV. Exercises. Derive the conclusions of each of the following arguments using the 8 Rules of Implication.

1.
1. $A \cdot B$
2. $(A \vee E) \supset D$ / $A \cdot D$

2.
1. $L \vee M$
2. $N \cdot \sim O$
3. $N \supset \sim L$ / M

3.
1. $(P \cdot \sim\sim Q) \supset R$
2. $\sim S \supset P$
3. $\sim Q \supset S$
4. $\sim S \cdot T$ / R

4.
1. $I \supset J$
2. $J \supset K$
3. $L \supset M$
4. $I \vee L$ / $K \vee M$

5.
1. $(U \supset T) \cdot (W \supset X)$
2. $U \cdot Y$
3. Z / $(T \vee X) \cdot Z$

6.
1. $G \supset (\sim H \cdot I)$
2. $H \vee J$
3. G / $J \vee K$

7.
1. $(A \vee B) \supset F$
2. $(F \vee B) \supset [A \supset (D \equiv E)]$
3. $A \cdot D$ /$D \equiv E$

8. If either Sandy or Ted win, then both Ulalume and Vicky lose. Sandy wins. Prove that Ulalume loses.

9. If Will once beat the fireman at billiards, then Will is not the fireman. If the brakeman is Xavier, then Xavier is not the fireman. If Will is not the fireman, and Xavier is not the fireman, then Yolanda is the fireman. If the brakeman is Xavier and Yolanda is the fireman, then Will is the engineer. Will once beat the fireman at billiards. The brakeman is Xavier. Prove that Will is the engineer.

Solutions may vary.

V. Sample solution for Exercise 9

Using the following legend:
A: Will once beat the fireman at billiards
B: Will is the fireman
C: The brakeman is Xavier
D: Xavier is the fireman
E: Yolanda is the fireman
F: Will is the engineer

So:

1. A ⊃ ~B	
2. C ⊃ ~D	
3. (~B · ~D) ⊃ E	
4. (C · E) ⊃ F	
5. A	
6. C	/F
7. ~B	1, 5, MP
8. ~D	2, 6, MP
9. ~B · ~D	7, 8, Conj
10. E	3, 9 MP
11. C · E	6, 10, Conj
12. F	4, 11, MP

QED