The Right Logic? Handout

I. Branching quantifiers

1. Some relative of each villager and some relative of each townsman hate each other.

2. (x){Vx ⊃ (∃y){Ryx • (z)[Tz ⊃ (∃w)(Rwz • Hyw • Hwy)]}}

3. (x){Tx ⊃ (∃y){Ryx • (z)[Vz ⊃ (∃w)(Rwz • Hyw • Hwy)]}}

4. (x)(∃y)(z)(∃w)[(Vx ⊃ Ryx) • (Tz ⊃ Rwz) • Hyw • Hwy]

5.   (x)(∃y) :
                : [(Vx ⊃ Ryx) • (Tz ⊃ Rwz) • Hyw • Hwy]
         (z)(∃w) :

6. Some book by every author is referred to in some essay by every critic.

II. Virtues of first-order logic

1. Completeness
2. A variety of definitions of logical truth concur: in terms of logical structure, substitution of
      sentences or of terms, satisfaction by models, and proof.
3. Every consistent first-order theory has a model.
4. Compactness
5. Upward and downward Löwenheim-Skolem features

All of these properties fail in second-order logic; see Mendelson, *Introduction to Mathematical Logic*, p
377.

III. Change of logic - change of subject

Chair$_1$: desk chairs, dining room chairs, and such, but not recliners or bean bag chairs
Chair$_2$: all chair$_1$ objects, and also recliners and bean bag chairs

IV. The existence of God

|1. ~(∃x)x=g   Assumption, for indirect proof
|2. (x)x=x   Principle of identity
|3. (x)~x=g   1, Change of quantifier rule
|4. g=g   2, UI
|5. ~g=g   3, UI
|6. g=g • ~g=g   4, 5, Conj
6. (∃x) x=g   1-5, Indirect proof