Philosophy 240: Symbolic Logic
Fall 2009

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Second-Order Logic Handout

I. Second-order quantification

1. $(\exists x)(Rx \bullet Ax)$
2. $(\exists x)(Rx \bullet Fx)$
3. $Ra \bullet Rb$
4. $(\exists X)(Xa \bullet Xb)$
5. No two distinct things have all properties in common.

6. Identical objects share all properties.

7. The identity of indiscernibles

8. $(X)(X \vee {\sim}X)$
9. Cat is to meow as dog is to bark.
10. $(\exists X)(Xcm \bullet Xdb)$
11. $(P)\{\{Nz \bullet Pz \bullet (x)[(Nx \bullet Px) \supset Pf(x)]\} \supset (x)(Nx \supset Px)\}$

II. Formation rules for S

Vocabulary of S

Capital letters
- A...U, used as predicates
- V, W, X, Y, and Z, used as predicate variables

Lower case letters
- a, b, c, d, e, i, j, k...u are used as constants.
- f, g, and h are used as functors.
- v, w, x, y, z are used as variables.

Five connectives: ~, •, ∨, ⊃ ≡

Quantifier: ∃

Punctuation: (), [], {}

Formation rules for wffs of S.

1. An n-place predicate or predicate variable followed by n terms (constants, variables, or functor terms) is a wff.
2. If α is a wff, so are
   $(\exists x)\alpha$, $(\exists y)\alpha$, $(\exists z)\alpha$, $(\exists w)\alpha$, $(\exists v)\alpha$
   $(x)\alpha$, $(y)\alpha$, $(z)\alpha$, $(w)\alpha$, $(v)\alpha$
3. If α is a predicate variable, and β is a wff, then the following are wffs
   $(\exists\alpha)\beta$, $(\alpha)\beta$
4. If α is a wff, so is ~α.
5. If α and β are wffs, then so are:
   $(\alpha \cdot \beta)$, $(\alpha \vee \beta)$, $(\alpha \supset \beta)$, $(\alpha \equiv \beta)$
6. These are the only ways to make wffs.

III. More sample translations

12. Everything has some relation to itself.

13. All people have some property in common.

14. No two people have every property in common.

15. Reflexivity: (x)Rxx
16. Symmetry: (x)(y)(Rxy ⊃ Ryx)
17. Transitivity: (x)(y)(z)[(Rxy • Ryz) ⊃ Rxz]
18. Some relations are transitive.

19. Some relations are symmetic, while some are asymmetric.

20. x=y iff (X)(Xx ≡ Xy)

IV. Higher-Order Logics

21. All useful properties are desirable.
    (X)(***U***X ⊃ ***D***X)

22. A man who possesses all virtues is a virtuous man, but there are virtuous man who do not possess all virtues:
    (x){[Mx • (X)(***V***X ⊃ Xx)] ⊃ ***V***x} • (∃x)[Mx • Vx • (∃X)(***V***X • ~Xx)]

23. There are at least two distinct properties.
24. (∃X)(∃Y)X≠Y
25. (∃X)(∃Y)(∃x)~Xx≡Yx

V. Second-order logic and set theory

26. (∃x)(Bx. • Hx)
27. (∃X)(∃x)(∃y)(Px • Py • x≠y • Xx • Xy)
28. P ∨ ~P