Philosophy 240: Symbolic Logic
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Functions Handout

I. An argument

1. No odd numbers are even.
2. One is odd.
3. One is the square of one. / So, not all square numbers are even.

1. (x)(Ox ⊃ ~Ex)
2. Oo

3. (∃x)[Sxo • (y)(Syo ⊃ y=x) • x=o]

But, we will explore another option.

II. Some functions, and their logical representations:

the father of:	f(x)
the successor of:	g(x)
the sum of:	f(x,y)
the teacher of:	$g(x_1...x_n)$

III. These are not functions:

the parents of a
the classes that a and b share
the square root of x

IV. Vocabulary of **FF**

Capital letters A...Z, used as one-place predicates
Lower case letters
- a, b, c, d, e, i, j, k...u are used as constants.
- f, g, and h are used as functors.
- v, w, x, y, z are used as variables.

Five connectives: ~, •, ∨, ⊃ ≡
Quantifiers: ∃, ∀
Punctuation: (), [], {}

An n-tuple of terms is a series of terms (constants, variables, or functor terms) separated by commas.
Some n-tuples.

a,b
a,a,f(a)
x,y,b,d,f(x),f(a,b,f(x))
a

If α is an n-tuple of terms, then the following are all functor terms:

f(α)
g(α)
h(α)

Some complex funtions:

f(f(x))
f(g(x))
h(h(h(x)))

V. Formation rules for wffs of **FF**

1. An n-place predicate followed by n terms (constants, variables, or functor terms) is a wff.
2. If α is a wff, so are
   (∃x)α, (∃y)α, (∃z)α, (∃w)α, (∃v)α
   (x)α, (y)α, (z)α, (w)α, (v)α
3. If α is a wff, so is ~α.
4. If α and β are wffs, then so are:
   (α · β)
   (α ∨ β)
   (α ⊃ β)
   (α ≡ β)
   By convention, you may drop the outermost brackets.
6. These are the only ways to make wffs.

VI. Some sentences of **FF**

Olaf loves his mother:	Log(o)
Olaf loves his grandmothers:	Log(g(o)) • Log(f(o))
Noone is his/her own mother:	(x)~x=g(x)

VII. Peano's Axioms for Arithmetic (following Mendelson, with adjustments)

P1: 0 is a number
P2: The successor (x') of every number (x) is a number
P3: 0 is not the successor of any number
P4: If x'=y' then x=y
P5: If P is a property that may (or may not) hold for any number, and if
   i. 0 has P; and
   ii. for any x, if x has P then x' has P;
   then all numbers have P.

VIII. Translations into **FF**

o: one
f(x): the successor of x
f(x, y): the product of x and y
Ex: x is even
Ox: x is odd
Px: x is prime

1. One is not the successor of any number.

2. If the product of a pair of numbers is odd, then the product of the successors of those numbers is even.

3. There are no prime numbers such that their product is prime.

IX. The original argument, redux

1. (x)(Ox ⊃ ~Ex)	
2. Oo	
3. o=f(o)	/ ~(x)Ef(x)
4. Of(o)	2, 3, ID
5. Of(o) ⊃ ~Ef(o)	1, UI
6. ~Ef(o)	5, 4, MP
7. (∃x)~Ef(x)	6, EG
8. ~(x)Ef(x)	7, CQ

QED