Philosophy 240: Symbolic Logic
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Modal Logic Handout

1. It is not the case that the sun is shining.
2. It is possible that the sun is shining.
3. It is necessary that this sun is shining.

4. Formation rules for propositional modal logic
   A single capital English letter is a wff.
   If α is a wff, so is ~α.
   If α and β are wffs, then so are (α · β), (α ∨ β), (α ⊃ β), and (α ≡ β).
   If α is a wff, then so are ◊α and □α.
   These are the only ways to make wffs.

5. □α :: ~◊~α
6. ◊α :: ~□~α

7. Leibniz's argument that this is the best of all possible worlds
   L1. God is omnipotent so he can create the best possible world.
   L2. God is omni-benevolent, so he wants to create the best possible world.
   L3. The world exists.
   LC1. So, this is the best of all possible worlds.
   LC2. Corollary: All of the evil in this world is necessary.

8. Actual World semantics
   V(~α) = ⊤ if V(α) = ⊥; otherwise V(~α) = ⊥
   V(α • β) = ⊤ if V(α) = ⊤ and V(β) = ⊤; otherwise V(α • β) = ⊥
   V(α ∨ β) = ⊤ if V(α) = ⊤ or V(β) = ⊤; otherwise V(α • β) = ⊥
   V(α ⊃ β) = ⊤ if V(α) = ⊥ or V(β) = ⊤; otherwise V(α ⊃ β) = ⊥
   V(α ≡ β) = ⊤ if V(α) = V(β); otherwise V(α ≡ β) = ⊥

9. Some sample propositions
   P: The penguin is on the TV.
   Q: The cat is on the mat.
   R: The rat is in the hat.
   S: The seal is in the sea.

   In w_1, we'll take P, Q, R, and S all to be true.

10. Translate each of the following claims into English, and determine their truth values, given the translation key and truth assignments above.
    1. P ⊃ Q
    2. P ⊃ R
    3. ~(R ∨ S)
    4. (P • Q) ⊃ R
    5. (Q ∨ ~R) ⊃ ~P

11. Semantics, indexed for alternate worlds

$V(\sim\alpha, w_n) = \top$ if $V(\alpha, w_n) = \bot$; otherwise $V(\sim\alpha, w_n) = \bot$

$V(\alpha \bullet \beta, w_n) = \top$ if $V(\alpha, w_n) = \top$ and $V(\beta, w_n) = \top$; otherwise $V(\alpha \bullet \beta, w_n) = \bot$

$V(\alpha \vee \beta, w_n) = \top$ if $V(\alpha, w_n) = \top$ or $V(\beta, w_n) = \top$; otherwise $V(\alpha \vee \beta) = \bot$

$V(\alpha \supset \beta, w_n) = \top$ if $V(\alpha, w_n) = \bot$ or $V(\beta, w_n) = \top$; otherwise $V(\alpha \supset \beta, w_n) = \bot$

$V(\alpha \equiv \beta, w_n) = \top$ if $V(\alpha, w_n) = V(\beta, w_n)$; otherwise $V(\alpha \equiv \beta, w_n) = \bot$

12. A small universe:

$U = \{w_1, w_2, w_3\}$

At w_1, P, Q, R and S are all true.

At w_2, P and Q are true, but R and S are false.

At w_3, P is true, and Q, R, and S are false.

13. Determine the truth values of each of the following claims at w_2 and w_3.

1. $P \supset Q$
2. $P \supset R$
3. $\sim(R \vee S)$
4. $(P \bullet Q) \supset R$
5. $(Q \vee \sim R) \supset \sim P$

14. Translate each of the following claims into English, and determine their truth values

1. $P_1 \supset (P_2 \bullet P_3)$
2. $S_1 \bullet S_2 \bullet S_3$
3. $R_1 \vee R_2 \vee R_3$
4. $[(P_1 \vee P_2) \supset (Q_1 \vee Q_2)] \bullet (P_3 \supset \sim Q_3)$

15. Leibnizian possible world semantics

$V(\Box\alpha, w_n) = \top$ if $V(\alpha, w_n) = \top$ for all w_n in U

$V(\Box\alpha, w_n) = \bot$ if $V(\alpha, w_n) = \bot$ for any w_n in U

$V(\Diamond\alpha, w_n) = \top$ if $V(\alpha, w_n) = \top$ for any w_n in U

$V(\Diamond\alpha, w_n) = \bot$ if $V(\alpha, w_n) = \bot$ for all w_n in U

16. Determine the truth values of each of the following propositions, given the semantics and assignments above.

1. $\Box(P \supset Q)_1$
2. $\Diamond(P \supset Q)_1$
3. $\Box P_1 \supset \Box Q_1$
4. $\Diamond P_1 \supset \Diamond Q_1$
5. $\Diamond[(Q \vee \sim R) \supset \sim P]_1$
6. $\Diamond P_1 \supset [Q_1 \supset \Box(R \bullet S)_1]$
7. Which of the truth values of the above sentences vary if considered at w_2 or w_3 (i.e. if we replace all the subscripts with '2's or '3's)?

17. Two types of questions

Metaphysical

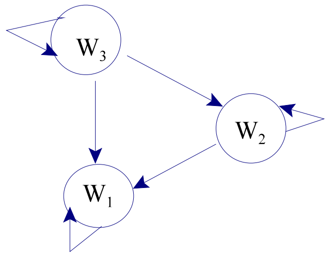
- What is the nature of a possible world?
- Do they exist?
- Are they abstract objects?
- Are they other states of this world, or are they independent of us?

Epistemological

- How do we know about possible worlds?
- Do we stipulate them, or do we discover them?
- Do we learn about them by looking at our world?
- Do we learn about them by pure thought?

18. Accessibility relations

$R = \{<w_1, w_1>, <w_2, w_1>, <w_2, w_2>, <w_3, w_1>, <w_3, w_2>, <w_3, w_3>\}$



19. Possible world semantics (Kripkean)

$V(\Box\alpha, w_n) = \top$ if $V(\alpha, w_m) = \top$ for all w_m in U such that $<w_n, w_m>$ is in R

$V(\Box\alpha, w_n) = \bot$ if $V(\alpha, w_m) = \bot$ for any w_m in U such that $<w_n, w_m>$ is in R

$V(\Diamond\alpha, w_n) = \top$ if $V(\alpha, w_m) = \top$ for any w_m in U such that $<w_n, w_m>$ is in R

$V(\Diamond\alpha, w_n) = \bot$ if $V(\alpha, w_m) = \bot$ for all w_m in U such that $<w_n, w_m>$ is in R

20. Determine the truth values of each of the following formulas, using Kripkean semantics, and given the accessibility relation in 18.

1. $\Box(P \supset Q)_1$
2. $\Box(P \supset Q)_3$
3. $\Diamond{\sim}(Q \vee R)_1$
4. $\Diamond{\sim}(Q \vee R)_2$
5. $\Diamond{\sim}(Q \vee R)_3$
6. $\Box P_1 \supset \Box Q_1$
7. $\Box P_3 \supset \Box Q_3$

21. System K

K: $\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$

(Nec) α / $\Box\alpha$

(Reg) $\alpha \supset \beta$ / $\Box\alpha \supset \Box\beta$

22. System D, deontic logic

D: $\Box\alpha \supset \Diamond\alpha$

Not provable in D: $\Box\alpha \supset \alpha$

23. Epistemic logic, and S4

Hintkka's epistemic logic takes three axioms:

K: $\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$

T: $\Box\alpha \supset \alpha$

4: $\Box\alpha \supset \Box\Box\alpha$

Any logic with the T axiom will have a reflexive accessibility relation.

Any logic with the 4 axiom will also have a transitive accessibility relation.

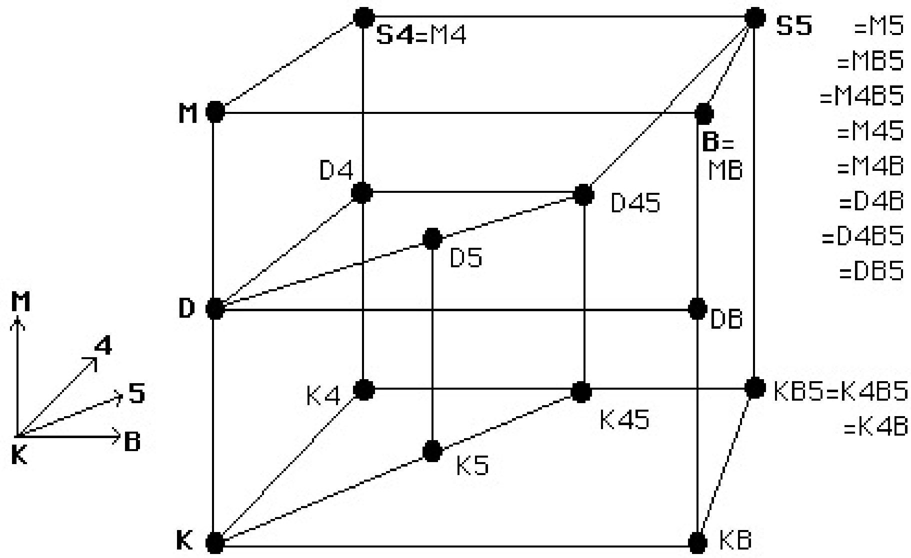
A system with all three of these axioms is called S4.

24. S5

S5 takes K, T, 4, and:

B: $\alpha \supset \Box\Diamond\alpha$

25. Relations among different modal systems



26. A criticism of modal logic

A Nine is greater than seven.
B: The number of planets is greater than seven.

C: The number of planets = nine.

D: Necessarily, nine is greater than seven.
E: Necessarily, the number of planets is greater than seven.