Philosophy 240: Symbolic Logic
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Translation Using Relational Predicates II Handout

1. $(x)[Px \supset (y)(Py \supset Lxy)]$
2. $(x)(y)[(Px \cdot Py) \supset Lxy]$
3. $(y)(x)[(Px \cdot Py) \supset Lxy]$

4. $(\exists x)[Px \cdot (\exists y)(Py \cdot Lxy)]$
5. $(\exists x)(\exists y)[(Px \cdot Py) \cdot Lxy]$
6. $(\exists y)(\exists x)[(Px \cdot Py) \cdot Lxy]$

7. Everyone loves someone: $(x)(\exists y)[Px \supset (Py \cdot Lxy)]$
8. Everyone is loved by someone: $(x)(\exists y)[Px \supset (Py \cdot Lyx)]$
9. Someone loves everyone: $(\exists x)(y)[Px \cdot (Py \supset Lxy)]$
10. Someone is loved by everyone: $(\exists x)(y)[Px \cdot (Py \supset Lyx)]$

7'. $(x)[Px \supset (\exists y)(Py \cdot Lxy)]$
8'. $(x)[Px \supset (\exists y)(Py \bullet Lyx)]$
9'. $(\exists x)[Px \cdot (y)(Py \supset Lxy)]$
10'. $(\exists x)[Px \cdot (y)(Py \supset Lyx)]$

11. $(x)[(\exists y)Lxy \supset Hx]$
12. $(x)(\exists y)(Lxy \supset Hx)$

Rules of passage

RP1:	$(\exists x)(Fx \lor Gx)$	::	$(\exists x)Fx \lor (\exists x)Gx$
RP2:	$(x)(Fx \bullet Gx)$	::	$(x)Fx \bullet (x)Gx$
RP3:	$(\exists x)(\alpha \bullet Fx)$	::	$\alpha \bullet (\exists x)Fx$
RP4:	$(x)(\alpha \bullet Fx)$	::	$\alpha \bullet (x)Fx$
RP5:	$(\exists x)(\alpha \lor Fx)$	::	$\alpha \lor (\exists x)Fx$
RP6:	$(x)(\alpha \lor Fx)$	::	$\alpha \lor (x)Fx$
RP7:	$(\exists x)(\alpha \supset Fx)$	::	$\alpha \supset (\exists x)Fx$
RP8:	$(x)(\alpha \supset Fx)$	::	$\alpha \supset (x)Fx$
RP9:	$(\exists x)(Fx \supset \alpha)$	::	$(x)Fx \supset \alpha$
RP10:	$(x)(Fx \supset \alpha)$	::	$(\exists x)Fx \supset \alpha$

13. $(\exists x)[Px \bullet (y)(Qy \supset Rxy)]$
14. $(\exists x)(y)[Px \bullet (Qy \supset Rxy)]$ by RP4

15. $(\exists x)(y)[Px \supset (Qy \supset Rxy)]$
16. $(\exists x)[Px \supset (y)(Qy \supset Rxy)]$ by RP8

12. (x)(∃y)(Lxy ⊃ Hx)
17. (x)[(y)Lxy ⊃ Hx]   by RP9

12. (x)(∃y)(Lxy ⊃ Hx)
18. (x)(∃y)(~Lxy ∨ Hx)   12, Impl
19. (x)(∃y)(Hx ∨ ~Lxy)   18, Com
20. (x)[Hx ∨ (∃y)~Lxy]   19, RP5
21. (x)[(∃y)~Lxy ∨ Hx]   20, Com
22. (x)[~(y)Lxy ∨ Hx]   21, CQ
23. (x)[(y)Lxy ⊃ Hx]   22, Impl   (Note that 23 is the same as 17.)

11. (x)[(∃y)Lxy ⊃ Hx]
24. (x)(y)(Lxy ⊃ Hx)   by RP10

25. (x)[Px ⊃ (∃y)Qy]
26. (∃x)Px ⊃ (∃y)Qy   by RP10

Metalogical proof of RP10
Consider first what happens when α is true, and then when α is false.

If α is true, then both formulas will turn out to be true.
The consequent of the formula on the right is just α.
So, if α is true, the whole formula on the right will be true.
‘ℱx ⊃ α’ will be true for every instance of x, since the consequent is true.
So, the universal generalization of each such formula (which is the formula on the left) will be true.
If α is false, then the truth value of each formula will depend.
To show that the truth values of each formula will be the same, we will show that the formula on the right is true in every case that the formula on the left is true and that the formula on the left is true in every case that the formula on the right is true.
If the formula on the left turns out to be true when α is false, it must be because ‘ℱx’ is false, for every x.
But then, ‘(∃x)ℱx’ will be false, and so the formula on the right turns out to be true.
If the formula on the right turns out to be true, then it must be because ‘(∃x)ℱx’ is false.
And so, there will be no value of ‘x’ that makes ‘ℱx’ true, and so the formula on the right will also turn out to be (vacuously) true.
QED

‘If anything was damaged, then everyone gets upset’:
27. (∃x)Dx ⊃ (x)(Px ⊃ Ux)
28. (x)[Dx ⊃ (y)(Py ⊃ Uy)]

§8.6: I.24, “If there are any cheaters, then if all referees are vigilant, they will be punished.”
29. (x){Cx ⊃ [(y)(Ry ⊃ Vy) ⊃ Px]}
29'. (x){Cx ⊃ (∃y)[(Ry ⊃ Vy) ⊃ Px]}   RP9
29". (x)(∃y){Cx ⊃ [(Ry ⊃ Vy) ⊃ Px]}   RP7

30: If there is a philosopher whom all philosophers contradict, then there is a philosopher who contradicts himself.

31 $(\exists x)[Fx \bullet (y)(Fy \supset Gyx)] \supset (\exists x)(Fx \bullet Gxx)$

32: $(\exists x)[Fx \bullet (y)(Fy \supset Gyx)] \supset (\exists z)(Fz \bullet Gzz)$

$(\exists z)(\exists x)\{[Fx \bullet (y)(Fy \supset Gyx)] \supset (Fz \bullet Gzz)\}$ by RP7

$(\exists z)(\exists x)\{(y)[Fx \bullet (Fy \supset Gyx)] \supset (Fz \bullet Gzz)\}$ by RP4

33: $(\exists z)(\exists x)(\exists y)\{[Fx \bullet (Fy \supset Gyx)] \supset (Fz \bullet Gzz)\}$ by RP9

$(x)\{[Fx \bullet (y)(Fy \supset Gyx)] \supset (\exists z)(Fz \bullet Gzz)\}$ by RP10

$(x)\{(y)[Fx \bullet (Fy \supset Gyx)] \supset (\exists z)(Fz \bullet Gzz)\}$ by RP4

$(x)(\exists y)\{[Fx \bullet (Fy \supset Gyx)] \supset (\exists z)(Fz \bullet Gzz)\}$ by RP9

34: $(x)(\exists y)(\exists z)\{[Fx \bullet (Fy \supset Gyx)] \supset (Fz \bullet Gzz)\}$ by RP7

33 and 34 are equivalent to 32 (and 31).
33 and 34 are both in prenex form.
But, they differ in form from each other.

35 $\vdash$ 36, but 36 $\sim\vdash$ 35.

35. $(\exists x)[Px \bullet (y)(Qy \supset Rxy)]$

36. $(\exists x)(y)[Px \supset (Qy \supset Rxy)]$

To see that 36 $\sim\vdash$ 35, we can construct a counter-example in a universe with two-members

36'. $(y)[Pa \supset (Qy \supset Ray)] \vee (y)[Pb \supset (Qy \supset Rby)]$

36''. $\{[Pa \supset (Qa \supset Raa)] \bullet [Pa \supset (Qb \supset Rab)]\} \vee \{[Pb \supset (Qa \supset Rba)] \bullet [Pb \supset (Qb \supset Rbb)]\}$

35'. $[Pa \bullet (y)(Qy \supset Ray)] \vee [Pb \bullet (y)(Qy \supset Rby)]$

35''. $[Pa \bullet (Qa \supset Raa) \bullet (Qb \supset Rab)] \vee [Pb \bullet (Qa \supset Rba) \bullet (Qb \supset Rbb)]$

To form the counter-example, just assign false to both ‘Pa’ and ‘Pb’.
Then, both conjuncts in 35'' are false, but all the conditionals in 36'' are (vacuously) true.

37. $(x)Fx \vee (x)Gx \vdash (x)(Fx \vee Gx)$

But 38. $(x)(Fx \vee Gx) \sim\vdash (x)Fx \vee (x)Gx$ To see 38, just substitute ‘P’ for F and ‘~P’ for G

39. $(\exists x)(Fx \bullet Gx) \vdash (\exists x)Fx \bullet (\exists x)Gx$

But 40. $(\exists x)Fx \bullet (\exists x)Gx \sim\vdash (\exists x)(Fx \bullet Gx)$

41. $(x)(Fx \supset \alpha) \vdash (x)Fx \supset \alpha$

But 42. $(x)Fx \supset \alpha \sim\vdash (x)(Fx \supset \alpha)$

43. $(\exists x)Fx \supset \alpha \vdash (\exists x)(Fx \supset \alpha)$

e.g. $(\exists x)Px \supset (\exists y)Qy \vdash (\exists x)[Px \supset (\exists y)Qy]$

But 44. $(\exists x)(Fx \supset \alpha) \sim\vdash (\exists x)Fx \supset \alpha$

e.g. $(\exists x)[Px \supset (\exists y)Qy] \sim\vdash (\exists x)Px \supset (\exists y)Qy$

Here are four logical truths of **F**.

45. $(y)[(x)Fx \supset Fy]$
46. $(y)[Fy \supset (\exists x)Fx]$
47. $(\exists y)[Fy \supset (x)Fx]$
48. $(\exists y)[(\exists x)Fx \supset Fy]$

Explore:

49. $(\exists x)(\alpha \equiv Fx)$
50. $\alpha \equiv (\exists x)Fx$
51. $(x)(\alpha \equiv Fx)$
52. $\alpha \equiv (x)Fx$

Translate each of the following sentences into predicate logic.

1. Everyone loves something. (Px, Lxy)
2. No one knows everything. (Px, Kxy)
3. No one knows everyone.
4. Every woman is stronger than some man. (Wx, Mx, Sxy: x is stronger than y)
5. No cat is smarter than any horse. (Cx, Hx, Sxy: x is smarter than y)
6. Dead men tell no tales. (Dx, Mx, Tx, Txy: x tells y)
7. There is a city between New York and Washington. (Cx, Bxyz: y is between x and z)
8. Everyone gives something to someone. (Px, Gxyz: y gives x to z)
9. A dead lion is more dangerous than a live dog. (Ax: x is alive, Lx, Dx, Dxy: x is more dangerous than y)
10. A lawyer who pleads his own case has a fool for a client. (Lx, Fx, Pxy: x pleads y’s case; Cxy: y is a client of x)