Philosophy 240: Symbolic Logic Fall 2009

## Predicate Logic Translation II Handout

I. Relational predicates: Lab: Andres loves Beatriz Gcde: Camila gave David the earring.

II. Languages that we have seen or will see this term:

PL: Propositional Logic
M: Monadic (First-Order) Predicate Logic
F: Full (First-Order) Predicate Logic
FF: Full (First-Order) Predicate Logic with functors
S: Second-Order Predicate Logic

## III. Vocabulary of M

Capital letters A...Z, used as one-place predicates Lower case letters

a, b, c,...u are used as constants. v, w, x, y, z are used as variables. Five connectives: ~, •, ∨, ⊃ ≡ Quantifiers: ∃, ∀ Punctuation: (), [], {} Constants and variables are called **terms**.

## IV. Scope and Binding

- 1.  $(x)(Px \supset Qx)$ Every P is Q2.  $(x)Px \supset Qx$ If everything is P, then x is QA formula is in the scope of a negation if either:
  - a. it directly follows the negation; or
    - b. what follows the negation is a bracket, and the formula occurs between the opening and closing of that bracket.
- The scope of the quantifier is whatever formula immediately follows the quantifier.
  - If what follows the quantifier is a bracket, then any formulas that occur until that bracket is closed are in the scope of the quantifier.
  - If what follows the quantifier is a negation, then every formula in the scope of the negation is in the scope of the quantifier.

Quantifiers bind every instance of their variable in their scope.

A **bound variable** is attached to the quantifier which binds it.

In 1, the 'x' in 'Qx' is bound.

In 2, the 'x' in 'Qx' is not bound.

An unbound variable is called a **free variable**.

The following open sentences contain both bound and free variables

- 3. (x)Px  $\lor$  Qx
- 4.  $(\exists x)(Px \lor Qy)$

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V. Formation rules for wffs of M

1. A predicate (capital letter) followed by a constant or variable (lower-case letter) is a wff.

2. If  $\alpha$  is a wff, so are

 $(\exists x)\alpha, (\exists y)\alpha, (\exists z)\alpha, (\exists w)\alpha, (\exists v)\alpha$  $(x)\alpha, (y)\alpha, (z)\alpha, (w)\alpha, (v)\alpha$ 

- 3. If  $\alpha$  is a wff, so is  $\sim \alpha$ .
- 4. If  $\alpha$  and  $\beta$  are wffs, then so are:
  - $(\alpha \cdot \beta)$

 $(\alpha \lor \beta)$ 

- $(\alpha \supset \beta)$
- $(\alpha \equiv \beta)$
- By convention, you may drop the outermost brackets.
- 5. These are the only ways to make wffs.

VI. A few more terms:

A wff constructed only using rule 1 is called an **atomic formula**.

A wff that is part of another wff is called a **subformula**.

Wffs that contain at least one unbound variable are called **open sentences**.

If a wff has no free variables, it is a **closed sentence**, and expresses a **statement**, or **proposition**.

Translations into M should yield closed sentences.

Quantifiers and connectives are called operators, or logical operators.

Note that atomic formulas lack operators.

The last operator added according to the formation rules is called the **main operator**.

## VII. Exercises

- 1. All mice are purple. (Mx, Px)
- 2. No mice are purple.
- 3. Some mice are purple.
- 4. Some mice are not purple.
- 5. Snakes are reptiles. (Sx, Rx)
- 6. Snakes are not all poisonous. (Sx, Px)
- 7. Children are present. (Cx, Px)

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- 8. Executives all have secretaries. (Ex, Sx)
- 9. Only executives have secretaries.
- 10. All that glitters is not gold. (Gx, Ax)
- 11. Nothing in the house escaped destruction. (Hx, Ex)
- 12. Blessed is he that considers the poor. (Bx, Cx)
- 13. Some students are intelligent and hard working. (Sx, Ix, Hx)
- 14. He that hates dissembles with his lips, and lays up deceit within him. (Hx, Dx, Lx)
- 15. Everything enjoyable is either illegal, immoral, or fattening. (Ex, Lx, Mx, Fx)
- 16. Some medicines are dangerous if taken in excessive amounts. (Mx, Dx, Tx)
- 17. Some medicines are dangerous only if taken in excessive amounts.
- 18. Victorian houses are attractive (Vx, Hx, Ax)
- 19. Slow children are at play. (Sx, Cx, Px)
- 20. Any horse that is gentle has been well-trained. (Hx, Gx, Wx)
- 21. Only well-trained horses are gentle.
- 22. Only gentle horses have been well-trained.
- 23. A knowledgeable, inexpensive mechanic is hard to find. (Kx, Ex, Mx, Hx)

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24. Dogs and cats chase birds and squirrels. (Dx, Cx, Bx, Sx)

25. If all survivors are women, then some women are fortunate. (Sx, Wx, Fx)

- 26. Some, but not all, of us got away. (Ux, Gx)
- 27. If all ripe bananas are yellow, then some yellow things are ripe. (Rx, Bx, Yx)
- 28. If any employees are lazy and some positions have no future, then some employees will not be successful. (Ex, Lx, Px, Fx, Sx)
- 29. No coat is waterproof unless it has been specially treated. (Cx, Wx, Sx)
- 30. A professor is a good lecturer if and only if he is both well-informed and entertaining. (Px, Gx, Wx, Ex)