Three-Valued Logics Handout

I. Motivations for three-valued logics

   A. Every even number greater than four can be written as the sum of two odd primes.
   B. There will be a party tomorrow night on the Dunham Quad.
   C. The king of America is bald.
   D: The king of America is not bald.
   E: The woman on the moon is six feet tall.
   F: The rational square root of three is less than two.
   G: When did you stop beating your wife?
   H. 'H' is false.
   I. 'Yields falsehood when appended to its own quotation' yields falsehood when appended to its
        own quotation.
   J. 'Heterological' is heterological.
   K. Quadruplicity drinks procrastination. (From Bertrand Russell)
   L. Colorless green ideas sleep furiously. (From Noam Chomsky)
   M. My minivan is a car.
   N. It is a nice day.
   O: 'O' is untrue.
   P: If 2+3-6 then pigs fly (and other paradoxes of material implication).

II. Exploring three-valued logics

   1. The rules for each;
   2. How the new rules affect the logical truths (tautologies); and
   3. How the new rules affect the allowable inferences (valid arguments).

III. Some exercises:

Construct truth tables for each of the following propositions, under classical semantics and each of the
three three-valued semantics (Bochvar, Kleene, Lukasiewicz). Compare the results. Note: these
exercises might form some part of a paper on three-valued semantics.

   1. P ∨ ~P
   2. P ⊃ P
   3. (P ⊃ Q) ≡ (~P ∨ Q)
        Note: you can construct the truth table for the biconditional by remembering that 'P≡Q'
             is logically equivalent to '(P ⊃ Q) • (Q ⊃ P)'