

Solutions to Homework Handout #6: Functions

- I.
1. $\sim Po \bullet Pf(o)$
 2. $(x)[Nx \supset (\exists y)y=f(x)]$
 3. $(x)(y)[(Ny \bullet Oy \bullet x=f(y)) \supset Ex]$
 4. $(x)(y)\{(Nx \bullet Ny) \supset \{Eg(x, y) \supset [(Ex \bullet Ey) \vee (Ox \bullet Oy)]\}\}$
 5. $(x)(y)\{(Nx \bullet Ny) \supset \{Og(x, y) \supset [(Ex \bullet Oy) \vee (Ox \bullet Ey)]\}\}$
 6. $(x)(y)[(Nx \bullet Px \bullet Ny \bullet Py) \supset \sim Pf(x, y)]$
 7. $(x)[(Nx \bullet Ex) \supset Eg(x)] \bullet (x)[(Nx \bullet Ox) \supset Og(x)]$
 8. $(x)[(Nx \bullet Ex) \supset Of(g(x))]$
 9. $(x)[(Nx \bullet Px \bullet \sim x=t) \supset Og(t, x)]$
 10. $(\exists x)(\exists y)[Nx \bullet Px \bullet Ny \bullet Py \bullet \sim x=y \bullet f(x, y)=f(g(x, y))]$

II. Sample derivations

1.

1. $(x)[Ax \vee Bf(x)]$	$/ (x)[Af(x) \vee Bf(f(x))]$
2. $Af(x) \vee Bf(f(x))$	1, UI
3. $(x)[Af(x) \vee Bf(f(x))]$	2, UG

QED

2.

1. $(x)(y)[f(x)=y \supset Cyxc]$	
2. $\sim Cf(a)ac$	$/ f(a) \neq b$
3. $f(a)=b$	AIP
4. $(y)[f(a)=y \supset Cyac]$	UI, 1
5. $f(a)=b \supset Cbac$	UI, 4
6. $Cbac$	5, 3, MP
7. $Cf(a)ac$	6, 3, ID
8. $Cf(a)ac \bullet \sim Cf(a)ac$	7, 2, Conj
9. $f(a) \neq b$	

QED

3.

1. $(x)\{Dx \supset (y)[\sim Exy \equiv Gf(f(y))]\}$	
2. $(x)(Dx \bullet \sim Gx)$	$/ (x)Ef(x)f(x)$
3. $Df(x) \bullet \sim Gf(x)$	2, UI
4. $Df(x) \supset (y)[\sim Ef(x)y \equiv Gf(f(y))]$	1, UI
5. $Df(x)$	3, Simp
6. $(y)[\sim Ef(x)y \equiv Gf(f(y))]$	4, 5, MP
7. $Df(f(x)) \bullet \sim Gf(f(x))$	2, EI
8. $\sim Ef(x)f(x) \equiv Gf(f(x))$	6, UI
9. $[\sim Ef(x)f(x) \supset Gf(f(x))] \bullet [Gf(f(x)) \supset \sim Ef(x)f(x)]$	8, Equiv
10. $\sim Ef(x)f(x) \supset Gf(f(x))$	9, Simp
11. $\sim Gf(f(x))$	7, Com, Simp
12. $Ef(x)f(x)$	10, 11, MT, DN
13. $(x)Ef(x)f(x)$	12, UG

QED