

Sample Solutions to Homework Handout #3 - Practice Problems for Test #3

1.
 1. $G \supset H$
 2. $G \supset I$ $/G \supset (H \cdot I)$
 3. G ACP
 4. H 1, 3, MP
 5. I 2, 3, MP
 6. $H \cdot I$ 4, 5, Conj
 7. $G \supset (H \cdot I)$ 3-6, CP

QED

2.
 1. $J \equiv K$
 2. $\sim K \equiv \sim L$ $/J \equiv L$
 3. $(J \supset K) \cdot (K \supset J)$ 1, Equiv
 4. $(\sim K \supset \sim L) \cdot (\sim L \supset \sim K)$ 2, Equiv
 5. $J \supset K$ 3, Simp
 6. $\sim L \supset \sim K$ 4, Com, Simp
 7. $K \supset L$ 6, Trans
 8. $J \supset L$ 5, 7, HS
 9. $\sim K \supset \sim L$ 4, Simp
 10. $L \supset K$ 9, Trans
 11. $K \supset J$ 3, Com, Simp
 12. $L \supset J$ 10, 11, HS
 13. $(J \supset L) \cdot (L \supset J)$ 8, 12, Conj
 14. $J \equiv L$ 13, Equiv

QED

3.
 1. M $/N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$
 2. N ACP
 3. O ACP
 4. P ACP
 5. $M \cdot N$ 1, 2, Conj
 6. $O \cdot P$ 3, 4, Conj
 7. $(M \cdot N) \cdot (O \cdot P)$ 5, 6, Conj
 8. $P \supset [(M \cdot N) \cdot (O \cdot P)]$ 4-7, CP
 9. $O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}$ 3-8, CP
 10. $N \supset \{O \supset \{P \supset [(M \cdot N) \cdot (O \cdot P)]\}\}$ 2-9, CP

QED

4.
 1. R ACP
 2. $R \supset S$ ACP
 3. S 2, 1, MP
 4. $(R \supset S) \supset S$ 2-3, CP
 5. $R \supset [(R \supset S) \supset S]$ 1-4, CP

QED

- 5.
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|-----|-------------------------|--------------|
| 1. | $D \supset E$ | |
| 2. | $E \supset (F \cdot G)$ | |
| 3. | $\sim F \vee \sim G$ | $/\sim D$ |
| | 4. D | AIP |
| | 5. E | 1, 4, MP |
| | 6. $F \cdot G$ | 2, 5, MP |
| | 7. F | 6, Simp |
| | 8. $\sim G$ | 3, 7, DN, DS |
| | 9. G | 6, Com, Simp |
| | 10. $G \cdot \sim G$ | 9, 8, Conj |
| 11. | $\sim D$ | 4-10, IP |

QED

- 6.
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|-----|--------------------------------------|----------------|
| 1. | $(P \vee Q) \vee (\sim R \vee S)$ | |
| 2. | $\sim Q \cdot (\sim S \cdot \sim P)$ | $/\sim R$ |
| 3. | $\sim Q$ | 2, Simp |
| 4. | $(Q \vee P) \vee (\sim R \vee S)$ | 1, Com |
| 5. | $Q \vee [P \vee (\sim R \vee S)]$ | 4, Assoc |
| 6. | $P \vee (\sim R \vee S)$ | 5, 3, DS |
| 7. | $\sim S \cdot \sim P$ | 2, Com, Simp |
| 8. | $\sim P$ | 7, Com, Simp |
| 9. | $\sim R \vee S$ | 6, 8, DS |
| 10. | $\sim S$ | 7, Simp |
| 11. | $\sim R$ | 9, 10, Com, DS |

QED

- 7.
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|----|--|--------------|
| | 1. $\sim \{A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]\}$ | AIP |
| | 2. $\sim A \cdot \sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1, DM |
| | 3. $\sim A$ | 2, Simp |
| | 4. $\sim [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 2, Com, Simp |
| | 5. $\sim [\sim A \vee (B \cdot C)]$ | 4, Dist |
| | 6. $A \cdot \sim (B \cdot C)$ | 5, DM, DN |
| | 7. A | 6, Simp |
| | 8. $A \cdot \sim A$ | 7, 3, Conj |
| 9. | $A \vee [(\sim A \vee B) \cdot (\sim A \vee C)]$ | 1-8, IP, DN |

QED

- 8.
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|---|---------------------|
| 1. $(A \vee B) \supset \sim(F \bullet D)$ | |
| 2. $\sim(A \bullet \sim D)$ | |
| 3. $\sim F \supset \sim(C \bullet D)$ | |
| 4. $C \vee A$ | / $A \equiv \sim C$ |
| 5. A | ACP |
| 6. $A \vee B$ | 5, Add |
| 7. $\sim(F \bullet D)$ | 6, 1, 6, MP |
| 8. $\sim F \vee \sim D$ | 7, DM |
| 9. $\sim A \vee \sim \sim D$ | 2, DM |
| 10. $\sim \sim D$ | 9, 5, DM, DS |
| 11. $\sim F$ | 8, 10, Com, DS |
| 12. $\sim(C \bullet D)$ | 3, 11, MP |
| 13. $\sim C \vee \sim D$ | 12, DM |
| 14. $\sim C$ | 13, 10, Com, DS |
| 15. $A \supset \sim C$ | 5-14, CP |
| 16. $\sim C$ | ACP |
| 17. A | 4, 16, DS |
| 18. $\sim C \supset A$ | 16, 17, CP |
| 19. $(A \supset \sim C) \bullet (\sim C \supset A)$ | 15, 18, Conj |
| 20. $A \equiv \sim C$ | 19, Equiv |

QED

- 9.
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|------------------------------|---------------|
| 1. $(A \bullet B) \supset C$ | |
| 2. $(F \bullet D) \supset E$ | |
| 3. $A \bullet D$ | |
| 4. $B \vee F$ | / $C \vee E$ |
| 5. $\sim(C \vee E)$ | AIP |
| 6. $\sim C \bullet \sim E$ | 5, DM |
| 7. $\sim C$ | 6, Simp |
| 8. $\sim(A \bullet B)$ | 1, 7, MT |
| 9. $\sim A \vee \sim B$ | 8, DM |
| 10. A | 3, Simp |
| 11. $\sim B$ | 9, 10, DN, DS |
| 12. F | 4, 11, DS |
| 13. D | 3, Com, Simp |
| 14. $F \bullet D$ | 12, 13, Conj |
| 15. E | 2, 14, MP |
| 16. $\sim E$ | 6, Com, Simp |
| 17. $E \bullet \sim E$ | 15, 16, Conj |
| 18. $C \vee E$ | 5-17, IP, DN |

QED

10. 1. $\sim F \supset (G \bullet H)$
 2. $F \supset (I \vee J)$
 3. $\sim [G \bullet (K \bullet \sim J)]$
 4. $K \equiv (L \bullet \sim J)$ / $K \supset I$
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|--|---|
| 5. K
6. $[K \supset (L \bullet \sim J)] \bullet [(L \bullet \sim J) \supset K]$
7. $K \supset (L \bullet \sim J)$
8. $L \bullet \sim J$
9. $\sim J$
10. $K \bullet \sim J$
11. $\sim G \vee \sim (K \bullet \sim J)$
12. $\sim (K \bullet \sim J) \vee \sim G$
13. $\sim G$
14. $\sim I$
15. $\sim I \bullet \sim J$
16. $\sim (I \vee J)$
17. $\sim F$
18. $G \bullet H$
19. G
20. $G \bullet \sim G$
21. I | ACP
4, Equiv
6, Simp
7, 5, MP
8, Com, Simp
5, 9, Conj
3, DM
11, Com
12, 10, DN, DS
AIP
14, 9, Conj
15, DM
2, 16, MT
1, 17, MP
18, Simp
19, 13, Conj
14-20, IP
5-21, CP |
|--|---|
22. $K \supset I$

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