Philosophy 240: Symbolic Logic — Hamilton College
Fall 2009 — Russell Marcus
Mondays, Wednesdays, Fridays: 9am - 9:50am — rmarcus1@hamilton.edu

Solutions to Practice Problems for Test #6

I. Translations

1. (x)(Fx ⊃ Px)
2. (x)[(Gx · Fx) ⊃ Px]
3. (x)(Fx ⊃ Mnx)
4. (x)(Wpx ⊃ Mnx)
5. (x)(Sxn ⊃ Mnx)
6. (x)(~Mxx ⊃ Mnx)
7. (x)(y)(Sxy ⊃ ~Syx)
8. (x){(Fx · Px) ⊃ (∃y)[(Gy · Py) · Rxy]} ⊃ (∃x)[Gx · (y)(Fy ⊃ Sxy)]
9. Bam · (x)(Bxm ⊃ x=a)
   (∃x)(Nx · Wex)
   ~(∃x)(Nx · Wax) / ~Bem
10. (x)(y)(Rxy ⊃ ~x=y) / (x)~Rxx
11. (x)(y)(z)(x=y ∨ y=z ∨ x=z)
    (∃x)(~x=c · Hx) / (∃x)(∃y)[~x=y · (z)(z=x ∨ z=y)]
12. (∃x)[Bxc · (y)(Byc ⊃ y=x) · Hx] / (∃x)(Bxc)
13. (x)[(~x=m · ~x=b) ⊃ Hx]
    ~Ha / a=m ∨ a=b
14. Ff(b) • ~Fg(c)
15. ~(∃x)Bxf(c,e)
16. Gc ⊃ {(Hg(c) • Rg(c)) • [Ff(c) • (∃x)(Nx • Wf(c)x)]}
17. (∃X)(Xn • ~Xp)
18. (x)(y)[(Rx • Ry) ⊃ (∃X)(Xx • Xy)]
19. (∃X){(x)(y)(z)[(Xxy • Xyz) ⊃ Xxz] • (x)(y)(Xxy ⊃ ~Xyx)}
20. (x)x=x / (∃X)Xxx

II. Derivations

Note: These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

1. 1. (x)(∃y)(~Fx ∨ Gy) / (x)Fx ⊃ (∃y)Gy
   | 2. (x)Fx ACP
   | 3. (∃y)(~Fx ∨ Gy) 1, UI
   | 4. ~Fx ∨ Ga 3, EI
   | 5. Fx 2, UI
   | 6. Ga 4, 5, DN, DS
   | 7. (∃y)Gy 6, EG
   8. (x)Fx ⊃ (∃y)Gy 2-7 CP

QED

2.

1. $(x)(\exists y)Fxy \supset (x)(\exists y)Gxy$		
2. $(\exists x)(y)\sim Gxy$	/ $(\exists x)(y)\sim Fxy$	
3. $\sim(x)(\exists y)Gxy$	2, CQ, CQ	
4. $\sim(x)(\exists y)Fxy$	1, 3, MT	
5. $(\exists x)(y)\sim Fxy$	4, CQ, CQ	

QED

3.

1. $(x)[(Fx \vee Gx) \supset (Hx \cdot Kx)]$		
2. $(x)\{(Hx \vee Lx) \supset [(Hx \cdot Nx) \supset Px]\}$	/ $(x)[Fx \supset (Nx \supset Px)]$	
| 3. Fx	ACP	
| 4. $Fx \vee Gx$	3, Add	
| 5. $(Fx \vee Gx) \supset (Hx \cdot Kx)$	1, UI	
| 6. $Hx \cdot Kx$	5, 4, MP	
| 7. Hx	6, Simp	
| 8. $Hx \vee Lx$	7, Add	
| 9. $(Hx \vee Lx) \supset [(Hx \cdot Nx) \supset Px]$	2, UI	
| 10. $(Hx \cdot Nx) \supset Px$	9, 8, MP	
| | 11. Nx	ACP	
| | 12. $Hx \cdot Nx$	7, 11, Conj	
| | 13. Px	10, 12, MP	
| 14. $Nx \supset Px$	11-13, CP	
15. $Fx \supset (Nx \supset Px)$	3-14, CP	
16. $(x)[Fx \supset (Nx \supset Px)]$	15, UG	

QED

4.

1. $\sim(\exists x)(Axa \cdot \sim Bxb)$		
2. $\sim(\exists x)(Dxd \cdot Dbx)$		
3. $(x)(Bex \supset Dxg)$	/ $\sim(Aea \cdot Dgd)$	
| 4. $Aea \cdot Dgd$	AIP	
| 5. $(x)\sim(Axa \cdot \sim Bxb)$	1, CQ	
| 6. $(x)(\sim Axa \vee Bxb)$	5, DM, DN	
| 7. $\sim Aea \vee Beb$	6, UI	
| 8 Aea	4, Simp	
| 9. Beb	7, 8, DN, DS	
| 10. $(x)\sim(Dxd \cdot Dbx)$	2, CQ	
| 11. $(x)(\sim Dxd \vee \sim Dbx)$	10, DM	
| 12. $\sim Dgd \vee \sim Dbg$	11, UI	
| 13. Dgd	4, Com, Simp	
| 14. $\sim Dbg$	12, 13, DN, DS	
| 15. $Beb \supset Dbg$	3, UI	
| 16. Dbg	15, 9, MP	
| 17. $Dbg \cdot \sim Dbg$	16, 14, Conj	
18. $\sim(Aea \cdot Dgd)$	4-17, IP	

QED

5.

1. (x)(Ax ⊃ Bx)		/ (x)[(∃y)(Ay · Cxy) ⊃ (∃z)(Bz · Cxz)]
	2. (∃y)(Ay · Cxy)	ACP
	3. Aa · Cxa	2, EI
	4. Aa	3, Simp
	5. Aa ⊃ Ba	1, UI
	6. Ba	5, 4, MP
	7. Cxa	3, Com, Simp
	8. Ba · Cxa	6, 7, Conj
	9. (∃z)(Bz · Cxz)	8, EG
10. (∃y)(Ay · Cxy) ⊃ (∃z)(Bz · Cxz)		2-9, CP
11. (x)[(∃y)(Ay · Cxy) ⊃ (∃z)(Bz · Cxz)]		10, UG

QED

6.

1. (∃x)(Nx · Wjx · Ix)	
2. Nc · Wjc · (x)[(Nx · Wjx) ⊃ x=c]	/ Ic
3. Na · Wja · Ia	1, EI
4. (x)[(Nx · Wjx) ⊃ x=c]	2, Com, Simp
5. (Na · Wja) ⊃ a=c	4, UI
6. Na · Wja	3, Simp
7. a=c	5, 6, MP
8. Ia	3, Com, Simp
9. Ic	8, 7, ID

QED

7.

1. (∃x){Mx · Tx · (y)[(My · y≠x) ⊃ Hxy]		/ (∃x){Mx · Tx · (y)[(My · ~Ty) ⊃Hxy]}
2. Ma · Ta · (y)[(My · ~y=a) ⊃ Hay]		1, EI
	3. My · ~Ty	ACP
	4. (y)[(My · ~y=a) ⊃ Hay]	2, Com, Simp
	5. (My · ~y=a) ⊃ Hay	4, UI
	| 6. y=a	AIP
	| 7. Ta	2, Simp
	| 8. ~Ty	3, Com, Simp
	| 9. Ty	7, 6, ID
	| 10. Ty · ~Ty	9, 8, Conj
	11. ~y=a	6-10, IP
	12. My	3, Simp
	13. My · ~y=a	12, 11, Conj
	14. Hay	5, 13, MP
15. (My · ~Ty) ⊃ Hay		3-14, CP
16. (y)[(My · ~Ty) ⊃ Hay]		15, UG
17. Ma · Ta		2, Simp
18. Ma · Ta · (y)[(My · ~Ty) ⊃ Hay]		17, 16, Conj
19. (∃x){Mx · Tx · (y)[(My · ~Ty) ⊃Hxy]}		18, EG

QED

8.

1. $(x)(y)(z)[(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (x=y \vee y=z \vee x=z)]$		
2. $(\exists x)(\exists y)(Sx \cdot Lx \cdot Sy \cdot Ly \cdot Rx \cdot Ry \cdot x \neq y)$		
3. $(x)(Rx \supset \sim Cx)$	/ $(Sa \cdot Ca) \supset \sim La$	
4. $Sa \cdot Ca$		ACP
5. La		AIP
6. $(\exists y)(Sb \cdot Lb \cdot Sy \cdot Ly \cdot Rb \cdot Ry \cdot b \neq y)$		3, EI
7. $Sb \cdot Lb \cdot Sc \cdot Lc \cdot Rb \cdot Rc \cdot b \neq c$		6, EI
8. $Sb \cdot Lb \cdot Sc \cdot Lc$		7, Simp
9. Sa		4, Simp
10. $Sa \cdot La$		9, 5, Conj
11. $Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc$		10, 8, Conj
12. $(y)(z)[(Sa \cdot La \cdot Sy \cdot Ly \cdot Sz \cdot Lz) \supset (a=y \vee y=z \vee a=z)]$		1, UI
13. $(z)[(Sa \cdot La \cdot Sb \cdot Lb \cdot Sz \cdot Lz) \supset (a=b \vee b=z \vee a=z)]$		12, UI
14. $(Sa \cdot La \cdot Sb \cdot Lb \cdot Sc \cdot Lc) \supset (a=b \vee b=c \vee a=c)$		13, UI
15. $a=b \vee b=c \vee a=c$		14, 11, MP
16. $\sim b=c$		7, Simp
17. $a=b \vee a=c$		17, 16, Com, DS
18. $Ra \supset \sim Ca$		3, UI
19. Ca		4, Com, Simp
20. $\sim Ra$		18, 19, DN, MT
21. Rb		7, Simp
22. $a=b$		AIP
23. $\sim Rb$		20, 22, ID
24. $Rb \cdot \sim Rb$		21, 24, Conj
25. $\sim a=b$		22-24, IP
26. $a=c$		17, 25, DS
27. Rc		7, Simp
28. $Rc \supset \sim Cc$		3, UI
29. $\sim Cc$		28, 27, MP
30. Cc		19, 26, ID
31. $Cc \cdot \sim Cc$		30, 29, Conj
33. $\sim La$		5-31, IP
34. $(Sa \cdot Ca) \supset \sim La$		4-33, CP

QED

9.

1. $(x)(y)f(x,y)=f(y,x)$		
2. $(x)f(x,o)=o$	/ $(x)f(o,x)=o$	
3. $f(x,o)=o$	2, UI	
4. $(y)f(o,y)=f(y,o)$	1, UI	
5. $f(o,x)=f(x,o)$	4, UI	
6. $f(o,x)=o$	4, 3, ID	
7. $(x)f(o,x)=o$	6, UG	

QED

10.

1. (x)(y)(Gxy ≡ Lyx)		
2. (x)Gf(x)x	/ (x)Lxf(x)	
3. (y)(Gf(x)y ≡ Lyf(x))	1, UI	
4. Gf(x)x ≡ Lxf(x)	3, UI	
5. Gf(x)x	2, UI	
6. Lxf(x)	4, 5, MP	
7. (x)Lxf(x)	6, UG	

QED

11.

1. (x)(y)(∃z)Sf(x)yz		
2. (x)(y)(z)[Sxyz ⊃ ~(Cxyz ∨ Mzyx)]	/ (∃x)(∃y)(∃z)~Mzg(y)f(g(x))	
3. (y)(∃z)Sf(g(x))yz		1, UI
4. (∃z)Sf(g(x))g(y)z		3, UI
5. Sf(g(x))g(y)a		4, EI
6. (y)(z)[Sf(g(x))yz ⊃ ~(Cf(g(x))yz ∨ Mzyf(g(x)))]		2, UI
7. (z)[Sf(g(x))g(y)z ⊃ ~(Cf(g(x))g(y)z ∨ Mzg(y)f(g(x)))]		6, UI
8. Sf(g(x))g(y)a ⊃ ~(Cf(g(x))g(y)a ∨ Mag(y)f(g(x)))		7, UI
9. ~(Cf(g(x))g(y)a ∨ Mag(y)f(g(x)))		8, 5, MP
10. ~Cf(g(x))g(y)a • ~Mag(y)f(g(x)))		9. DM
11. ~Mag(y)f(g(x)))		10, Com, Simp
12. (∃z)~Mzg(y)f(g(x)))		11, EG
13. (∃y)(∃z)~Mzg(y)f(g(x)))		12, EG
14. (∃x)(∃y)(∃z)~Mzg(y)f(g(x)))		13, EG

QED