

These solutions are merely samples. There are, for most problems, alternative, fully legitimate solutions.

1.
 1. $Ab \vee Bc$
 2. $(x)\sim Bx$ / $(\exists x)Ax$
 3. $\sim Bc$ 2, UI
 4. Ab 1,3 Com, DS
 5. $(\exists x)Ax$ 4, EG

2. 1. $(x)(Rx \supset Ox)$
2. $(\exists x)\sim Ox$
3. $(x)(\sim Rx \supset Px)$ / $(\exists x)Px$
4. $\sim Oa$ 2, EI
5. $Ra \supset Oa$ 1, UI
6. $\sim Ra$ 5, 4, MT
7. $\sim Ra \supset Pa$ 3, UI
8. Pa 7, 6, MP
9. $(\exists x)Px$ 8, EG

3.
 1. $(x)(Fx \supset Gx)$
 2. $(y)(Gy \supset Hy)$ / $(z)(\sim Hz \supset \sim Fz)$
 3. $Fx \supset Gx$ 1, UI
 4. $Gx \supset Hx$ 2, UI
 5. $Fx \supset Hx$ 3, 4, HS
 6. $\sim Hx \supset \sim Fx$ 5, Trans
 7. $(z)(\sim Hz \supset \sim Fz)$ 6, UG

4. 1. $(\exists x)(Ax \cdot Bx) \supset (x) Dx$
2. $\sim Da$ / $(x)(Ax \supset \sim Bx)$
3. $(\exists x)\sim Dx$ 2, EG
4. $\sim(x)Dx$ 3, CQ
5. $\sim(\exists x)(Ax \cdot Bx)$ 1, 4, MT
6. $(x)\sim(Ax \cdot Bx)$ 5, CQ
7. $(x)(\sim Ax \vee \sim Bx)$ 6, DM
8. $(x)(Ax \supset \sim Bx)$ 7, Impl

QED

5. 1. $(y)[Ay \supset (\sim By \supset Dy)]$
 2. $\sim Ba$ / $Aa \supset Da$
 | 3. Aa ACP
 | 4. $Aa \supset (\sim Ba \supset Da)$ 1, UI
 | 5. $\sim Ba \supset Da$ 4, 3, MP
 | 6. Da 5, 2, MP
 7. $Aa \supset Da$ 3-6, CP

QED

6. 1. $(x)(Qx \supset \sim Px)$ / $(\exists x)Px \supset \sim(x)Qx$
 | 2. $(\exists x)Px$ ACP
 | 3. Pa 2, EI
 | 4. $Qa \supset \sim Pa$ 1, UI
 | 5. $\sim Qa$ 4, 3, DN, MT
 | 6. $(\exists x)\sim Qx$ 5, EG
 | 7. $\sim(x)Qx$ 6, CQ
 8. $(\exists x)Px \supset \sim(x)Qx$ 2-7, CP

QED

7. 1. $(x)[Ax \supset (Bx \cdot Dx)]$
 2. $(x)[(Ax \cdot Dx) \supset Ex]$
 3. $(x)(Ex \supset \sim Dx)$ / $\sim Aa$
 | 4. Aa AIP
 | 5. $Aa \supset (Ba \cdot Da)$ 1, UI
 | 6. $Ba \cdot Da$ 5, 4, MP
 | 7. Da 6, Com, Simp
 | 8. $Aa \cdot Da$ 4, 7, Conj
 | 9. $(Aa \cdot Da) \supset Ea$ 2, UI
 | 10. Ea 9, 8, MP
 | 11. $Ea \supset \sim Da$ 3, UI
 | 12. $\sim Da$ 11, 10, MP
 | 13. $Da \cdot \sim Da$ 7, 12, Conj
 14. $\sim Aa$ 4-13, CP

QED

- 8.
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|---|---------------------------------------|
| 1. $(x)(Ax \supset Bx)$ | |
| 2. $(x)[Bx \supset (Ax \supset \sim Fx)]$ | |
| 3. $(x)[(\sim Cx \cdot Dx) \supset Fx]$ | $/ (x)[Ax \supset (Cx \vee \sim Dx)]$ |
| 4. Ax | ACP |
| 5. $Ax \supset Bx$ | 1, UI |
| 6. Bx | 5, 4, MP |
| 7. $Bx \supset (Ax \supset \sim Fx)$ | 2, UI |
| 8. $Ax \supset \sim Fx$ | 7, 6, MP |
| 9. $\sim Fx$ | 8, 4, MP |
| 10. $(\sim Cx \cdot Dx) \supset Fx$ | 3, UI |
| 11. $\sim(\sim Cx \cdot Dx)$ | 10, 9, MT |
| 12. $Cx \vee \sim Dx$ | 11, DM, DN |
| 13. $Ax \supset (Cx \vee \sim Dx)$ | 4-12, CP |
| 14. $(x)[Ax \supset (Cx \vee \sim Dx)]$ | 13, UG |

QED

- 9.
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|---|---------------|
| 1. $(\exists x)Gx \supset (x)(Fx \supset Dx)$ | |
| 2. $(\exists x)(Gx \cdot \sim Dx)$ | $/ \sim(x)Fx$ |
| 3. $Ga \cdot \sim Da$ | 2, EI |
| 4. Ga | 3, Simp |
| 5. $(\exists x)Gx$ | 4, EG |
| 6. $(x)(Fx \supset Dx)$ | 1, 5, MP |
| 7. $Fa \supset Da$ | 6, UI |
| 8. $\sim Da$ | 3, Com, Simp |
| 9. $\sim Fa$ | 7, 8, MT |
| 10. $(\exists x)\sim Fx$ | 9, EG |
| 11. $\sim(x)Fx$ | 10, CQ |

QED

- 10.
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|---|-------------------|
| 1. $(\exists x)Qx \supset (x)(Rx \supset Sx)$ | |
| 2. $(x)\sim Qx \supset (\exists x)Sx$ | |
| 3. $(x) Rx$ | $/ (\exists x)Sx$ |
| 4. $\sim(\exists x)Sx$ | AIP |
| 5. $\sim(x)\sim Qx$ | 2, 4, MT |
| 6. $(\exists x)Qx$ | 5, CQ |
| 7. $(x)(Rx \supset Sx)$ | 1, 6, MP |
| 8. $Rx \supset Sx$ | 7, UI |
| 9. Rx | 3, UI |
| 10. Sx | 8, 9, MP |
| 11. $(\exists x)Sx$ | 10, EG |
| 12. $(\exists x)Sx \cdot \sim(\exists x)Sx$ | 11, 4, Conj |
| 13. $(\exists x)Sx$ | 4-12, IP, DN |

QED

II. Invalidity

1. Invalid in a 1-member universe, where:

Aa: True; Ba: False; Ca: False.

2. Invalid in a 2-member universe, where:

Ea: True; Fa: False; Ga: False;

Eb: True or False; Fb: True; Gb: True

3. Invalid in a 3-member universe, where:

Pa: True; Qa: False; Ra: False

Pb: False; Qb: True; Rb: False

Pc: False; Qc: False; Rc: True or False