CHAPTER 1

Logic

Logic is *correct thinking*. In a wide variety of circumstances, without any formal training, we are able to do a good job of thinking over a range of complex problems.

So why study logic formally? First, there are systematic deficiencies in our untrained cognitive behavior that can to some extent be remedied by becoming aware of the deficiencies and aware of how we should in fact be reasoning. We will discuss many examples of this. Second, logic is part of the foundation of more advanced formal reasoning like mathematics. As we will see, logic provides us with a clear notion of proof. Having a rigorous concept of proof is critical to the advancement of mathematics. See *Proofs and Refutations* by Imre Lakatos, for a good historical example of how mathematicians struggled to prove Euler's conjecture because they didn't have a satisfactory notion of proof.

1. Arguments

An **argument** is attempt to convince someone of something. Arguments are attempts to give the listener or reader of the argument reasons for believing that some particular claim is true.

(1) There have to be reasons.
(2) There has to be some sort of presumption that the listener or reader does not already believe the particular claim.
(3) The reasons have to be intended for the listener or reader to come to believe the particular claim. They don't have to explain why the claim is true; they just have to give some reason why one should believe it.

How can you recognize an argument? One clue is that they have expressions like 'because', 'since', 'for', 'suppose', 'assume', 'therefore', 'hence', 'thus', 'it follows that', 'we can conclude that', 'for the following reasons'. But the clues are only a very rough guide and are often misleading. The definitive way to tell is to take all the clues available from the source and the context and to answer the question, "Does it give me reasons for adopting belief in some claim?"

1.1. The Difference between Arguments and Explanations. Often it is hard to tell the difference between and argument and an explanation because both use many of the key words like 'because', 'since', and 'so'.

Whether a written or spoken passage is an argument is *logically independent* of whether it is an explanation. This means: An argument might be an explanation and it might not.
An explanation might be an argument and it might not.

An explanation tells you why something is true.
An argument tells you why you should believe it.

Here is an explanation that is not an argument:
The sky is blue because the atmosphere is mainly composed of nitrogen, and nitrogen scatters blue light over a much wider range of angles than other colors.

Here is an argument that is not an explanation:
The sky is really red tonight; I've just been watching it.

Here is an argument that is an explanation:
The sky is usually clear where I live because I live on the east edge of the Rocky Mountains.

Here is a passage that is neither an argument nor an explanation:
The sky is cloudy. It gets cloudy here every now and then.

(1) Fats impair immune function. In both in-vitro tests and experiments using intravenous soybean oil infusions in volunteers, fats reduce the vigilance of white blood cells. Researchers in New York put healthy volunteers on diets that reduced fat content to 20 percent of calories. Three months later, blood samples showed that natural killer cell activity was greatly improved. Not surprisingly, vegetarians have been shown to have more than double the natural kill cell ability, compared to non-vegetarians.
(2) Among the many ways the pastoralists interact with and represent their aural environment, one stands out for its sheer ingenuity: a remarkable singing technique in which a single vocalist produces two distinct tones simultaneously. One tone is a low, sustained fundamental pitch, similar to the drone of a bagpipe. The second is a series of flutelike harmonics, which resonate high above the drone and may be musically stylized to represent such sounds as the whistle of a bird, the syncopated rhythms of a mountain stream or the lilt of a cantering horse. (Scientific American, September 1999)
(3) Under certain conditions, cow's milk proteins pass through the gut into the bloodstream, eliciting the production of antibodies. These antibodies end up attacking not only the milk proteins but also pancreatic beta-cell proteins that happen to be structurally similar to those in cow's milk. Viral infections cause these beta-cell proteins to be exposed to the antibodies. During viral infections over the next several years, intermittent antibody attacks gradually destroy the beta cells. In late childhood or early adulthood, insulin levels are so low that diabetes becomes manifest. (Karjalainen J, Martin JM, Knip M, et al. A bovine albumin peptide as a possible trigger of insulin-dependent diabetes mellitus. N Engl J Med 1992;327:302-7.)
(4) The idea that there is an epidemic of human cancer caused by synthetic industrial chemicals is false. Linear extrapolation from the near-toxic doses in rodents to low-level exposure in humans has led to grossly exaggerated mortality forecasts. Such extrapolations cannot be justified by epidemiological evidence. Furthermore, relying on such extrapolations for synthetic chemicals while ignoring the enormous natural background leads to an unbalanced perception of hazard and allocation of resources. Far from being the source of unhealthiness, the progress of scientific research and technology by industry will continue to lengthen human life expectancy; indeed, there has been a steady rise in life expectancy in the developed countries.

(5) It is one of the clearest symptoms of the decadence besetting the academy that the ideals that once informed the humanities have been corrupted, willfully misunderstood, or simply ignored by the new sophistries that have triumphed on our campuses. We know something is gravely amiss when teachers of the humanities confess–or, as is more often the case, when they boast–that they are no longer able to distinguish between truth and falsity. We know when something is wrong when scholars assure us–and their pupils–that there is no essential difference between the disinterested pursuit of knowledge and partisan proselytizing, or when academic literary critics abandon the effort to identify works of lasting achievement as a reactionary enterprise unworthy of their calling. And indeed, the most troubling development of all is that such contentions are no longer the exceptional pronouncements of a radical elite, but have increasingly become the conventional wisdom in humanities departments of our major colleges and universities.

(6) After spending more than $40 million on the investigation, the FBI and the National Transportation Safety Board (NTSB) have not found a definitive answer for why the center fuel tank exploded. Yet they have ruled out a missile as the cause. The NTSB believes an undetermined system flaw produced an electrical spark that ignited jet fuel vapors in the tank. Prior to the official embrace of this mechanical explanation, the missile expert was among several scientists invited by FBI agents to explore the missile theory. He was made privy to evidence suggesting that TWA 800 could have been shot down, consisting of eyewitness accounts of a "flare-like object" shooting skyward moments before the plane exploded. Later he examined the debris in the Calverton hangar. The missile expert has also been in contact with military labs where, he says, the chemists have been unable to make jet fuel vapor explode as the NTSB says it did in TWA 800's center fuel tank. "The labs told the NTSB there's a big problem–it can't happen." The NTSB wouldn't listen. He says, "They were adamant that [the labs] had to find something." Davey, R.; Village Voice; July 14-20, 1999.

(7) Today's public relations (PR) industry is related to democracy in the same way that prostitution is related to sex. When practiced voluntarily for love, both can exemplify human communications at its best. When they are bought and sold, however, they are transformed into something hidden and sordid. There is nothing wrong with many of the techniques used by the PR industry–lobbying, grassroots organizing, using the news media to put ideas before the public. As individuals, we not only have the right to engage in these activities, we have a responsibility to participate in the decisions that shape our society and our lives. Ordinary citizens have the right to organize for social change–better working conditions, health care, fair prices for family farmers, safe food, freedom from toxins, social justice, a humane foreign policy. But ordinary citizens cannot afford the multi-million dollar campaigns that PR firms undertake on the behalf of their special interest clients, usually large corporations, business associations and governments. Raw money enables the PR industry to

mobilize private detectives, attorneys, broadcast faxes, satellite feeds, sophisticated information systems and other expensive, high-tech resources to outmaneuver, overpower and outlast citizen reformers.

(8) In rather the same way as new movies are now 'reviewed' in terms of their first weekend gross, new candidates have become subject to evaluation by the dimensions of their 'war chest.' This silly, archaic expression defines other equally vapid terms like 'credibility' and 'electability' and 'name recognition,' which become subliminally attached to it. In many cases the crude cash-flow measure is as useful in deciding on a politician as it is in making a choice at the multiplex; you might as well see the worthless movie that everyone else has seen, or express an interest in the unbearably light 'front runner,' so as not to be left out of the national 'conversation.' (Hitchens C, The Nation, August 23/30 1999)

(9) Secondly, we must proceed to a definition of the term 'art'. This comes second, and not first, because no one can even try to define a term until he has settled in his own mind a definite usage of it: no one can define a term in common use until he has satisfied himself that his personal usage of it harmonizes with the common usage. Definition necessarily means defining one thing in terms of something else; therefore, in order to define any given thing, one must have in one's head not only a clear idea of the thing to be defined, but an equally clear idea of all the other things by reference to which one defines it.

(10) One of the most cherished freedoms in a democracy is the right to freely participate in the "marketplace of ideas." We value this freedom because without it, all our other freedoms are impossible to defend. In a democracy every idea, no matter how absurd or offensive, is allowed to compete freely for our attention and acceptance. Turn on the TV, and you'll find plenty of absurd and offensive examples of this principle in action. On the Sunday public affairs shows you'll find Republicans, Democrats, Republicans who love too much, and Democrats who love Republicans. On "A Current Affair" or "Oprah Winfrey," you'll find self-proclaimed werewolves, worshippers of Madonna, and doomsday prophets from the lunatic fringes of American society.

(11) The curse of having too many recreational options, of course, is further encouraged by the Dominican Republic's substantial development of large modern, luxurious all-inclusive resorts, scattered up and down both coastlines. Most offer nearly all watersports (from boardsailers and sailboats to wave runners and scuba diving), meals, alcoholic and non-alcoholic beverages, entertainment and children's daycare for incredibly low package rates. Couple this with the unusually low cost and affordable rates most airlines provide to the island and the Dominican Republic is perhaps one of the Caribbean's best kept secrets.

(12) According to David Hume, causes are sufficient conditions for their effects: "We may define a cause to be an object, followed by another, and where all the objects similar to the first, are followed by objects similar to the second." (1748, section VII.) Later writers refined Hume's theory, but still characterized the causal relation in terms of necessary and sufficient conditions. One of the best known approaches is Mackie's theory of inus

conditions. An inus condition for some effect is an insufficient but non-redundant part of an unnecessary but sufficient condition. Suppose, for example, that a lit match causes a forest fire. The lighting of the match, by itself, is not sufficient; many matches are lit without ensuing forest fires. The lit match is, however, a part of some constellation of conditions that are jointly sufficient for the fire. Moreover, given that this set of conditions occurred, rather than some other set sufficient for fire, the lighting of the match was necessary: without it, the fire would not have occurred.

(13) Beliefs produced by visual experience are in large part self-ascriptive: the subject believes not only that the world is a certain way but also that he himself is situated in the world in a certain way. To believe that the scene before my eyes is stormy is the same as to believe that I am facing a stormy part of the world. Elsewhere I have argued that the objects of such beliefs should be taken, and that the objects of all beliefs may be taken, as properties which the subject self-ascribes. Hence the content of visual experience likewise consists of properties–properties which the subject will self-ascribe if the visual experience produces its characteristic sort of belief.

(14) Video poker began as an afterthought, but as profits surged through the 1980's, operators longed for legitimacy. South Carolina law forbade games of chance, so at first the operators relied on linguistic chicanery: Poker, they said, equals pinball. Play pinball well enough and you win a free game. Same thing in video poker. Suppose it costs twenty-five cents to play a game of pinball or poker. Well, then each skillfully won free game must be worth twenty-five cents. And if each game is worth twenty-five cents, surely you ought to be able to collect a quarter for it. And if you can collect a quarter for one free game, surely you should be able to collect 4,000 quarters for drawing a royal flush. (David Plotz, Harpers, August 1999, p.66)

(15) The arrow of time is one of the deepest paradoxes of conventional physics today. According to all the laws of physics there should be no distinction between past and future, no direction to time. Since the second law of thermodynamics says that entropy necessarily increases with time, and thus the past and future differ, the second law, too, is contradicted.

(16) As new research is pushing forward the day the planet became habitable, other discoveries are pushing back the first signs of life. Microfossils found in ancient rocks from Australia and South Africa demonstrate that terrestrial life was certainly flourishing by 3.5 billion years ago. Even older rocks from Greenland, 3.9 billion years old, contain isotopic fingerprints of carbon that could have belonged only to a living organism. In other words, only 100 million years or so after the earliest possible point when Earth could have safely supported life, organisms were already well enough established that evidence of them remains today. This narrowing window of time for life to have emerged implies that the process might have required help from space molecules. (Scientific American, July 1999)

(17) On September 21, 1993, President Clinton signed into law the National and Community Service Trust Act, which appropriated $1.5 billion for an embryonic national service program. The Clinton national service plan

> will create 20,000 federally funded, full-time youth jobs this fall. That number will increase to 33,000 in 1995 and 47,000 in 1996, for a total of 100,000 participants. The lure for enrollment in the program consists of a $4,725 college scholarship for each year of service. The CNCS did not spring full-blown from the brow of Bill Clinton. The corporation is an enhancement of the 1990 National and Community Services Act, through which the states were encouraged to develop federally approved guidelines for "volunteer" community service through public schools and other agencies. As he signed the 1990 legislation, then-President George Bush declared, "From now on, any definition of a successful life must include serving others." The 1990 act earmarked $25 million for the creation of "peer-to-peer pressure groups" intended to goad reluctant citizens to "volunteer" for service .... However, Americans continue to be the world's most devoted altruists, donating considerable volunteer service without any prompting from government; this has been acknowledged by Eli Segal. In a speech made last November 8th, Segal pointed out that "94 million American citizens right now are doing volunteer work. Sixty percent of young people between 12 and 17 are now volunteering over three hours a week." The spontaneous private response to recent disasters, such as the Midwest flood and the California earthquake, illustrate that Americans are capable of addressing community needs without government intrusion. (Grigg, W.N.; The New American; Vol. 10, No. 06, March 21, 1994)

2. Sentences and Statements

There are many kinds of sentences:

(1) You're in danger.
(2) Who's there?
(3) Stop!
(4) I'd like a snack.
(5) Please.
(6) Silence is golden.
(7) Free your slaves, or else!
(8) I don't know.
(9) Vermont used to be a relatively fun place to go skiing, even if it isn't the cheapest site around.
(10) Huh?
(11) If I don't steal his watch, I doubt someone else will.

Any sentence of which it makes sense to say that it is true or it is false is called a **statement**. Statements are declarative sentences; they make some claim.

The statements are 1, 4, 6, 8, 9, 11. The others are not.

3. Answers to the Identification of Arguments

'A' means it is an argument, 'N' means it is not.

1. A, 2. N, 3. N, 4. A, 5. A, 6. N, 7. A, 8. N, 9. A, 10. A, 11. A, 12. N, 13. N, 14. N (It is a story reporting someone else's argument), 15. A, 16. A, 17. N.

4. Propositions

Werner said, "Ich hoffe du bist nicht krank." I report that Werner stated that he hopes you aren't sick. The content of Werner's claim is that he hopes you aren't sick. The two statements, "Ich hoffe du bist nicht krank," and "I hope you aren't sick," are two different sentences and two different statements that express the same content.

A **proposition** is the content of a statement. For example, Werner was expressing the proposition that he hopes you aren't sick. A proposition is not a language dependent entity, although it might be that some statements in some languages pick out propositions that cannot be (simply) expressed in other languages.

5. Truth

In classical logic, every statement has exactly one of the following two truth values: true or false.

There are no cases where it is both true and false. There are no cases where it fails to have a truth value. There are no in-betweens. There are no degrees of truth where for example we say something is mostly true or 98% true.

We will work with these assumptions throughout our discussion. In more advanced logic, we can relax some of these requirements and explore the philosophical consequences, but for now, we'll leave these issues to folks with a lot of time on their hands.

6. Vagueness

How do we evaluate whether a statement is true or false if it isn't clear cut?

We assume that there are some standards for judging that are as precise as one needs, and we evaluate the statement relative to one of these standards.

EXAMPLE 1. *"Madonna is old."*
"White is a color."

For the purposes of evaluating arguments, we almost never need to say that one standard is wrong and another one is right. We will almost always be able to say, you can go either way: it's either true or false depending on how you look at it.

7. Possibility

When a statement is possibly true, we say the statement is called **consistent**.

When a statement is not possibly true, we say the statement is **inconsistent**, or we say it's a **contradiction**.

EXAMPLE 2. *Consistent statements*
"There are only 3 planets in the Solar system."
"Copper is metallic."

We often say things like "I can't speak Portuguese," or "Birds can't fly to the moon." Yet statements that seem literally true in one sense, can be false in another sense.

In logic, we focus on the broadest sense of possibility. We take nothing for granted except being logically consistent. If we can coherently imagine it, it is possible.

I can coherently imagine that birds can fly to the moon, so it's logically possible for birds to fly to the moon.

EXAMPLE 3. *Inconsistent statements*

- *There are square circles.*
- *Colorless green bricks exist.*
- *Some mothers have had no children.*
- *Joe is from Manhattan, and he isn't from Manhattan.*
- *Fred Flintstone is just a fictional character, yet I know that he is a real live person.*

Rules for judging possibility: "If you can imagine it, it's possible, but you have to be imagining correctly (i.e. coherently).

In judging possibility, you have to be consistent in your standards for resolving vagueness. If the sentence has the word 'blue' at the beginning and 'blue' at the end, you have to have the same standards for determining what counts as blue.

You have to stick to the meanings that are given in natural language (like English). You can't say that it's possible for there to be round squares because of the fact that we could make up a language where the word 'round' means pepperoni and the word 'squares' means pizzas. Stick to what the English words mean, and then look for flexibility that allows the sentence to be true.

You cannot coherently imagine a mathematical or logical contradiction. You may think that you can imagine the following equation is true, but you cannot *coherently* imagine it:

$$(129442093) + (390 \times 1023) - 389920 = 399 \times (890013) - 219$$

8. Subtleties with Contradictions

There are a number of sentences that look like contradictions, but are not really contradictions in the strictest sense. Here are several that look contradictory but are not, or are at least not clearly contradictory.

(1) I don't think I exist.
(2) This flower is a peony, but I don't know whether it's a peony.
(3) The word 'horses' does not refer to horses.
(4) Nothing exists.
(5) Every sentence is true.
(6) If the coin is made of gold, then it isn't.
(7) Everyone is mortal, and everyone is immortal.
(8) There exists a blue object that has no physical extension.
(9) This statement I am making right now is not true.
(10) This sentence is not written in English.

9. Necessity

A statement that cannot be false is called **necessarily true** or just **necessary**, and we also call it a **tautology**.

A statement that can be true and can be false is called **contingent**.

EXERCISE 1. *Label each of the following as being necessary, contingent, or inconsistent*

Premise 1
Premise 2
Premise 3
...
Premise 62
Conclusion

(1) *Suddenly, all the stars in the nighttime sky disappeared, leaving only the moon visible to the naked eye or the astronomer's telescope.*
(2) *The princess kissed the frog, and it turned into a handsome goat.*
(3) *No human beings exist.*
(4) *There is a human being taller than every human being.*
(5) *The triangle had four sides, two of which were parallel.*
(6) *Napoleon proved that the Earth is flat.*
(7) *The astronaut lost his helmet during a space walk, but he survived just fine without any air to breathe.*
(8) *The daughter of my brother is my sister's nephew.*
(9) *Each point on a circle is equidistant from the center.*
(10) *Ashley was unemotional yet at the same time enraged.*
(11) *Godzilla stomped all over Tokyo, and then he turned himself into a delicious slice of cheesecake.*
(12) *I drink, but I don't exist.*
(13) *The archbishop was assassinated, but he hasn't died yet.*
(14) *There is a pain in my leg that I can't feel.*
(15) *She saw the invisible wizard.*
(16) *Kendrick saw the tiger escape the cage, but it didn't really escape.*

10. Formal Arguments

Formally, arguments are a finite number of statements, which we call **premises**, plus a single statement, which we call the **conclusion**.

We write arguments in the following way:

It never matters what order the premises are in.

11. Counterexample

A **counterexample** to an argument is a possible situation where all the premises are true and the conclusion is false.

All reptiles are green.
None of the animals in the zoo are reptiles.
____
There are no green animals in the zoo.

The counterexample must be a fully fleshed out situation where it is clear that all reptiles are green and that none of the animals in the zoo are reptiles while at the same time there are some green animals (or at least one) in the zoo.

EXERCISE 2. *Describe counterexamples to the following arguments*

Lenny has been coughing a lot lately.
____
Lenny is sick.

Charlotte has 3 children.
Charlotte has 3 daughters.

Frogs are aquatic.
Anything that lives in a pond or river is aquatic.
Frogs live in ponds or rivers.

12. Validity

VALID means NO COUNTEREXAMPLES
A **valid argument** is an argument that has no counterexamples.
An **invalid argument** is an argument that's not valid.

EXERCISE 3. *Identify which of the following arguments are valid.*

All philosophers are nerds.
Oscar is a philosopher.
Oscar is a nerd.

No creatures live on the moon.
The moon is inhabitable.

Some cats are white.
Fluffy is a cat.
Fluffy is white.

Bob has a red shirt.
Bob has a shirt that is colored.

Seth is unemployed.
Seth does not have a regular paying job.

All drummers own some kind of drum kit.
Jim owns his own drum kit.
Jim is a drummer.

Radishes are partially red.
Radishes are partially green.
Radishes are partially colored.

Oscar owns 3 Tool shirts.
3 Tool shirts belong to Oscar.

Only people do social work.
All social work is done by people.

No one who studied philosophy would have done poorly on the LSAT.
John is a person.
John did poorly on the LSAT.
John did not study philosophy.

Anyone who smokes will get lung cancer.
Trevor smokes.
Trevor will get cancer.

Not everyone who studies will get an A.
Lisa did not study.
Lisa will not get an A.

Amy had $300 in her piggy bank yesterday.
No one has come near her piggy bank within the last few days.
Amy still has $300 in her piggy bank.

Rhonda was born exactly 13 years ago.
Rhonda is alive today.
Rhonda has been alive for 13 years.

President Sadat was murdered.
President Sadat died.

New Delhi is a part of India.
India is a part of Asia.
New Delhi is a part of Asia.

Seth's brother is Steve.
Seth's son is John.
John is Steve's nephew.

$200 is too much to pay for an old laptop.
Jerry paid $160 for his old laptop.
Jerry didn't pay too much for his laptop.

All fruit trees produce flowers sometime during the year.
If I own a fruit tree, it will produce flowers sometime.

Dancing is popular in Turkmenistan.
Turkmenistan is in Asia.
Dancing is popular in Asia.

All actors are self-centered.
Groucho Marx was an actor.
Groucho Marx was self-centered.

The deadliest fish in the ocean is Oscar, the buck-toothed gar.
Sharks are not the deadliest fish in the ocean.

Oscar likes Tool.
Tool plays alt-metal music.
Oscar likes alt-metal music.

Paul either got a speeding ticket or he spent time in jail for reckless driving.
It turns out that Paul didn't spend any time in jail.
Thus, Paul got a speeding ticket.

Yesterday, Gus loaded up his truck with a load of manure at the feed lot, drove it home, dumped the w
Thus, Gus drove his truck yesterday.

One of the cattle got out of the pen yesterday afternoon.
No one at the ranch went out to find the cow and put her back in the pen.
Thus, there is one less cow in the pen this morning than there was yesterday morning.

All children like to eat cookies.
Amber Evans likes to eat cookies.
Thus, Amber Evans is a child.

The Colorado River runs through the Grand Canyon.
Kelli Hilgenfeld is rafting in the Colorado River.
Thus, Kelli Hilgenfeld is in the Grand Canyon.

13. Summary of Definitions

A **statement** is the proposition (or content) of a sentence that makes a claim.

A **proposition** is the content of a statement, what the statement claims.

A **necessary** proposition, or **tautology**, is a proposition that cannot possibly be false.

A **contradiction** is a proposition that cannot possibly be true.

A **contingent** proposition is one that is possibly true and possibly false.

A set of propositions is **consistent** if they can all be true together, i.e. if there is at least one possibility where all the propositions are true.

A set of propositions is **inconsistent** if they cannot all be true together.

An argument is a set of propositions (called **premises**) and a proposition (called a **conclusion**).

A **counterexample** to an argument is a possible situation where the premises are all true and the conclusion is false.

An argument is **valid** if it has no counterexamples.

An argument is **invalid** if it has at least one counterexample.

14. Why Logic is Tough

Your innate sense of which arguments are good and which ones are bad do not correspond with what logic says is valid and invalid. For instance, your brain is set up to think using important background information and to ignore the kind of outlandish possibilities that you need to consider when checking validity. The good news is that your informal reasoning ability is very good at most tasks, and with many tasks your innate ability surpasses the ability of any computer using the formal methods we will learn. The bad news is that many times our intuitive reasoning ability misleads us, including situations that are very important to us, like whether to have a risky surgery, how to design a multi-million dollar experiment.

The point to keep in mind is that there are tools of logic that can overcome the failings of our intuitive logic. The most difficult thing for you as a student of logic is to remember to use the rules. It is very tempting to take shortcuts, but dangerous as well.

14.1. Why Should We Care about Validity? In diagnosing a real argument, it is very useful to distinguish the strength of an argument's reasoning from the strength of its premises. An argument might start off making assumptions that everyone agrees are reasonable and well founded but then draw conclusions through fallacious reasoning. An argument might also use impeccable reasoning, but nevertheless be faulty because it starts off with outlandish assumptions.

Validity is the ultimate in **reasoning strength**. It gives us a guarantee that the conclusion is true if the premises are. The ultimate in **premise strength** is truth, although it often happens that we do not have reliable access to the truth. Thus, we often measure the strength of the premises in terms of their plausibility, which quantifies how likely a statement is to be true given all we know. Together, truth of the premises, and validity of the argument, constitute the ultimate in argument strength because they guarantee that the conclusion is true.

A valid argument is good because it gives us a 100% foolproof guarantee that if the premises are true, then the conclusion is also true.

There is one and only one burglar.
Only the burglar could have left the glass-cutter in the vault.
Gonzo the night watchman is the only person who could have left a glass-cutter in the vault.
Thus, Gonzo is the burglar.

Because this argument is valid, one way to prove that Gonzo burgled is to prove that each of the three premises is true. Once each of the premises has been proven true, logic dictates that Gonzo is the guilty party.

Likewise, if we can identify an argument as having poor reasoning, we know that it will be a waste of time to investigate whether the premises are true.

Our experiment showed that the people we tested who had heart attacks had higher IQ's than people we tested who didn't.
If IQ's don't have anything to do with heart health, then there is only a 0.4% chance that we would have gotten the results we got.
Thus, there is a 99.6% chance that people who have had heart attacks have higher IQ's than people who didn't have heart attacks.

Because this argument is invalid, we should not (on the basis of this argument) waste our time and money trying to repeat the experiment to verify the results.

14.2. VALID ARGUMENT $\neq$ GOOD ARGUMENT. An argument that is valid has superb reasoning strength, but that doesn't mean the argument is good. The overall argument strength also depends on whether the premises are true and several other factors we have not discussed.

Here is a valid argument that is not a good argument for two reasons:

Trees never need water to grow.
Thus, trees never need water to grow.

14.3. INVALID ARGUMENT $\neq$ BAD ARGUMENT. An invalid argument fails to give us a guarantee that the conclusion will be true when the premises are true, but sometimes we don't need an absolute guarantee. Almost all real arguments are invalid arguments.

Here is an invalid argument that is a pretty good argument:

Every emerald that has been found so far has been green.
Thus, the next emerald someone finds will be green.

14.4. Conversational vs. Logical Implication. Suppose someone says, "My preacher hasn't been drunk at all for this entire week!" What is the reasonable thing to infer?

One reasonable thing to infer is something like, "My preacher is often drunk."

This is not a logical inference because

My preacher hasn't been drunk at all for this entire week!
My preacher is often drunk.

is invalid. We call this kind of inference 'conversational' because the reason we find it plausible has to do with accepted rules for having conversations.

One rule for having conversations is that you don't normally restrict the information you give without having some reason for it. If my preacher seldom or never drinks, I wouldn't have any reason to mention that he hasn't been drunk for the past week. Therefore, when I do mention it, you are led to think that there must be a good reason for my specifying his being sober during the past week. The only good reason that comes to mind off-hand for my mentioning it seems to be that it's unusual, that he is normally drunk.

The way to tell a logical inference from a conversational inference is to see whether you can deny the inference (cancel it) without retracting what you originally said.

I can say, "My preacher hasn't been drunk this entire week. In fact, he never drinks." without having to retract anything.

I can't say, "My notebook is red, but it doesn't have any color." If I say it doesn't have any color, I am effectively retracting my comment that it is red.

15. EX FALSO QUODLIBET

This latin phrase means, "From an absurdity, anything follows." It is a consequence of the definition of validity is that if the premises are contradictory, the argument is valid, no matter what the conclusion is. Here's why:

1. Assume we have some inconsistent premises.
2. What it means for the premises to be inconsistent is that it is impossible for the premises to be all true.
3. If it is impossible for the premises to be all true, then it is impossible for the premises to be all true while the conclusion is false.
4. That means there are no counterexamples.
5. That means the argument is valid.

Godzilla does not really exist.
Geoff is not tall.
Geoff is tall.
____
Thus, Godzilla really exists.

$1 + 1 = 3$
____
Thus, the moon is made of cheese.

It is another consequence of the definition of validity that if the conclusion is tautologous, the argument is valid. Here's why:

1. Assume we have some tautologous conclusion.
2. What it means for the conclusion to be tautologous is that it is impossible for the conclusion to be false.
3. If it is impossible for the conclusion to be false, then it is impossible for the conclusion to be false while the premises are all true.
4. That means there are no counterexamples.
5. That means the argument is valid.

Trampolines are fun.
____
Thus, red is not the same color as blue.

16. Quiz

Definition: An argument is sound when (and only when) it is valid and all its premises are true.

(1) If an argument has all true premises,
a) it is invalid.
b) it is valid.
c) it is sound.
d) none of the above.

(2) You have come across an argument for which there is a possible world where all the premises are true and the conclusion is true. What can you conclude from this?
a) the argument is valid.
b) the argument is good.
c) the argument is sound.
d) none of the above.

(3) If there is no possibility where all the premises are true, then the argument will be a) invalid.
b) valid and sound.
c) a contradiction.
d) valid and unsound.

(4) The conclusion of a sound argument
a) is true.
b) might be true or might be false.
c) is always valid.
d) is false in the possible worlds where the premises are all true.

True or False

(5) As a rule, if an argument has all false premises, it cannot be valid.
(6) As a rule, if an argument has one false premise, it cannot be valid.
(7) An argument can be valid even if it has a false conclusion.
(8) A valid argument always has a true conclusion whenever the premises are true.
(9) An invalid argument always has a false conclusion whenever the premises are true.
(10) As a rule, if an argument has all false premises and a false conclusion, then it has to be invalid.
(11) As a rule, if an argument has all false premises and a true conclusion, then it has to be invalid.
(12) As a rule, if an argument has all true premises and a false conclusion, then it cannot be valid.
(13) As a rule, if an argument has a false conclusion, then it cannot be valid.
(14) As a rule, if an argument has a contradictory conclusion, then it cannot be valid.
(15) As a rule, if an argument has all false premises and true conclusion, then it cannot be valid.
(16) An invalid argument always has a false conclusion whenever the premises are true.

(17) As a rule, a valid argument always has a false conclusion if one of its premises is false.
(18) A valid argument can have a false conclusion, but only if one of the premises is false.

Why are the following definitions of validity bad definitions?

A counterexample is the case that proves the conclusion is false.

A counterexample is when the premises are true and the conclusion is false.

A counterexample is a situation that proves that the argument is invalid.

A counterexample is a situation where the if the premises are true, the conclusion is false.

A counterexample to an argument is a possible situation where all the premises are true and the conclusion is false.

Why are the following definitions of 'validity' bad?

A valid argument is a case where the premises are true and the conclusion is true as well.

A valid argument is a case where the premises are true and therefore the conclusion has to be true.

A valid argument is an argument where if the premises are true, then the conclusion is true.

A valid argument is an argument where the premises are true and the conclusion can't be false.

An argument is valid only when it has no counterexamples.

An argument is valid if and only if it is not invalid.

17. Answers

(1) All philosophers are nerds... V
(2) No creatures live on the moon... I
(3) Some cats are white... I
(4) Bob has a red shirt... I
(5) Seth is unemployed... V
(6) All drummers own some kind of drum kit... I
(7) Radishes are partially red... V
(8) Oscar owns 3 Tool shirts... V
(9) Only people do social work... V
(10) No one who studied philosophy... V
(11) Anyone who smokes will get lung cancer... V
(12) Not everyone who studies will get an A... I
(13) Amy had $300 in her piggy bank yesterday... I
(14) Rhonda was born exactly 13 years ago... I
(15) President Sadat was murdered... V
(16) New Delhi is a part of India... V
(17) Seth's brother is Steve... V
(18) $200 is too much to pay for an old laptop... I
(19) All fruit trees produce flowers sometime during the year... V
(20) Dancing is popular in Turkmenistan... I
(21) All actors are self-centered... V (If we interpret 'All actors are self-centered in a tenseless way, i.e. as a rule about all actors at all times, including those of the past. Otherwise, I.)

(22) The deadliest fish in the ocean is Oscar... Unclear answer (Ambiguous use of 'fish'.)
(23) Oscar likes Tool... I
(24) Paul either got a speeding ticket... V
(25) Yesterday, Gus loaded up his truck... V
(26) One of the cattle got out of the pen... I
(27) All children like to eat cookies... I
(28) The Colorado River runs through the Grand Canyon... I

Answers to the validity quiz: 1. d, 2. d, 3. d, 4. a, 5. F, 6. F, 7. T, 8. T, 9. F, 10. F, 11. F, 12. T, 13. F., 14. F, 15. F, 16. F, 17. F, 18. F.

CHAPTER 2

Propositional Logic

In this chapter, we will construct the logic. That is, we will define a formal **language** with rules for assigning natural language sentences to symbols in the formal language, and techniques for determining the validity of arguments, consistency of sentences, etc. Propositional logic is designed to capture more or less the underlying logic of various sentential connectives, i.e. combinatorial relations among parts of sentences that can be interpreted as propositional.

1. Argument Forms

Large classes of arguments share the same features. We can exploit this by using **argument forms**.

By dealing with the form of the argument we can deal with all the arguments that share that form. This will permit us to take a shortcut. Sometimes we won't have to stretch our imaginations thinking of possible counterexamples. Instead we can just look at the overall structure of the argument to tell whether it is valid.

EXERCISE 4. *Which of the following arguments are valid?*

> Brad went to school, and Jill stayed home sick.
> ___
> Thus, Brad went to school.

> Horses usually weigh over 800 lbs. and they have to eat a lot to maintain their weight.
> ___
> Thus, Horses usually weigh over 800 lbs.

> The Tralfamadorians offered an ulkin to the Earthlings, and then took one of the Earthlings away for experiment.
> ___
> Thus, the Tralfamadorians offered an ulkin to the Earthlings.

Any argument of the form

> *α and β.*
> ___
> *Thus, α.*

is a valid argument.

An **argument form** is a generic type of argument, a kind of argument skeleton. It usually contains some English words and some Greek letters.

Here is an argument form:

> *Because δ, it isn't so that γ.*
> ___
> *Thus, γ.*

Here is another:

> *It is crazy to believe that β.*
> *It is possible that ϵ.*
> ___
> *Thus, it is likely that ϵ, but α.*

When we fill in the Greek letters with statements, we get an argument.

Here we can fill in consistently to make an argument:

Let $\delta =$ "Everybody was able to see the screen."
Let $\gamma =$ "Yolanda couldn't sit up straight."

> *Because Yolanda couldn't sit up straight, it isn't so that everybody was able to see the screen.*
> *Thus, everybody was able to see the screen.*

Here is our other argument form filled in to make an argument:

Let $\beta =$ "NASCAR is fun to watch."
Let $\epsilon =$ "Spinach is my favorite food."
Let $\alpha =$ "I think I'm going to become a druid."

> *It is crazy to believe that NASCAR is fun to watch.*
> *It is possible that spinach is my favorite food.*
> *Thus, it is likely that NASCAR is fun to watch, but I think I'm going to become a druid.*

1.1. Fine Points on Argument Forms.

- Argument forms are constructed so that no matter what statements you plug in, the resulting sentences are grammatical and meaningful.
- Don't try to preserve the capitalization of the statements being substituted. Just make the finished argument have the correct capitalization.
- You have to be consistent when plugging in statements. If the same Greek letter appears in multiple parts of the argument form, you have to plug a single statement in for all instances of that Greek letter.
- An argument form when fully filled in will always yield an argument, but in general, the argument can be either valid or invalid.

Here is another argument form:

> *Unless α, β.*
> *Thus, α and β.*

Is the following argument an instance of this argument form?

> *Unless I feel like it, I'm not going swimming.*
> *Thus, I'm not going to the park, and I'm not going swimming.*

Here is the same argument form:

> *Unless α, β.*
> *Thus, α and β.*

The following *is* an instance of this argument form:

> *Unless I feel like it, I'm not going swimming.*
> *Thus, I feel like it, and I'm not going swimming.*

Let $\alpha =$ "I feel like it."
Let $\beta =$ "I'm not going swimming."

Here is another argument form:

> *α and β.*
> *δ.*
> *Thus, α and β.*

Is the following assignment of variables allowed?
Let $\alpha =$ "Jodi thinks I'm an idiot."
Let $\beta =$ "Trent thinks Jodi is right."
Let $\delta =$ "Jodi thinks I'm an idiot."

Jodi thinks I'm an idiot, and Trent thinks Jodi is right.
Jodi thinks I'm an idiot.
Thus, Jodi thinks I'm an idiot, and Jodi thinks I'm an idiot.

1.2. Valid Argument Forms. The reason we care about argument forms, is that there are some argument forms that are special. These special argument forms are constructed in such a way that no matter what statements are substituted in for the Greek letters, the resulting argument is valid. These special argument forms are called **valid argument forms**.

Every instance of a valid argument form is a valid argument.

Here are some valid argument forms:

α but β.
Thus, β.

α.
β.
Thus, α and β.

α or β.
It is not the case that α.
Thus, β.

If α, then β.
α.
Thus, β.

α unless β.
It is not the case that β.
Thus, α.

1.3. Invalid Argument Forms. An **invalid argument form** is a form that isn't valid. That means an invalid argument form has at least one instance that is an invalid argument.

For valid argument forms, all argument instances are valid.

For invalid argument forms, at least one argument instance is not valid.

Here is an invalid argument form:

α.
β.
Thus, δ.

Here is an invalid instance of this argument form. This argument proves that the form is invalid:

Let $\alpha =$ "Bill is a good snooker player."
Let $\beta =$ "Rust forms on old engine parts."
Let $\delta =$ There is gold in Alaska."

Bill is a good snooker player.
Rust forms on old engine parts.
Thus, there is gold in Alaska.

Here is the same invalid argument form:

α.
β.
Thus, δ.

Here is a valid instance of this argument form. Having a valid argument as an instance does not mean the argument form is valid!

Let $\alpha =$"Roses usually bloom in May."
Let $\beta =$ "Fireflies appear in June."
Let $\delta =$ "Roses usually bloom in May, and fireflies appear in June."

Roses usually bloom in May.
Fireflies appear in June.
Thus, roses usually bloom in May, and fireflies appear in June.

1.4. Summary on Argument Forms. To tell whether an argument form is invalid, fish around plugging in any statements you want until you get an invalid argument. If you find an invalid argument, the form is invalid. If you can't find an invalid argument, the form is valid. Strategy Tip: Just substitute random, unrelated statements into an argument form. If the argument form is invalid, it will usually reveal itself very quickly.

EXERCISE 5. *Mark the following argument forms with 'Valid' or 'Invalid'.*

α because β.
Thus, β.

α; however, β.
Thus, it isn't the case that α.

Most people believe that β.
Thus, β.

It isn't the case that α.
It isn't the case that β.
Thus, neither α nor β.

Rick is extremely confident that δ.
Thus, Rick believes that δ.

Either β or α.
β.
Thus, α.

β unless δ.
Thus, β.

α.
β.
δ.
Thus, α, β, and δ.

α or β.
δ
Thus, δ and β.

True or False Quiz

(1) As a rule, if an argument is invalid, then it is an instance of some invalid argument form.

(2) As a rule, if an argument is invalid, then it is an instance of some valid argument form.
(3) As a rule, if an argument is valid, then it is an instance of some invalid argument form.
(4) As a rule, if an argument is valid, then it is an instance of some valid argument form.

2. The Formal Language of Propositional Logic

We are going to develop a logic that can deal with the four following logical connections between sentences:
Conjunction: 'and'
Disjunction: 'or'
Negation: 'not'
Conditional: 'If. . . then. . . '

The good thing about propositional logic is that it simplifies arguments for us.

The argument

The cat is in the hat, and the dog is in the fog.
Thus, the cat is in the hat.

gets translated into the argument

C&D.
Thus, C.

where C is "The cat is in the hat," and D is "The dog is in the fog."

The benefit is that after breaking down complicated arguments into their component sentences (or clauses) and a handful of logical connectives like 'and', we'll be able to do some simple manipulations that will tell us with certainty whether the argument is valid. We won't have to worry about overlooking potential counterexamples.

2.1. Limitations of Propositional Logic. Because the logic only deals with the kinds of logical relationships that exist between simple sentences and compound sentences, there are many aspects of logic that the logic mishandles.

The following arguments are valid, but will appear to be invalid if we use propositional logic because none of the sentences are compound sentences using 'and' or 'or' or 'not' or 'if. . . then. . . '.

The gong was heard making a loud sound.
Thus, the gong was audible.

Jane is a grandmother.
Thus, Jane at some point had a child.

Wendy is a bank teller.
All bank tellers work at some or other bank.
Thus, Wendy works at a bank.

The way to translate into propositional logic is to rearrange the premises and conclusion so that they look like sentences (clauses) connected with the following expressions:
'and'
'it is not the case that'

‘or’
‘if then’

3. Conjunction

Conjunction is just the fancy name for ‘and’. To translate sentences that use the word ‘and’, we first rewrite them so that they have stand-alone clauses connected with the word ‘and’.

Haley and Rachel have a birthday next week.
Haley has a birthday next week, and Rachel has a birthday next week.

Kendrick bought some chocolate bars and some panty hose today.
Kendrick bought some chocolate bars today, and Kendrick bought some panty hose today.

Victoria is going to register her car, inflate her tires, and change her oil.
Victoria is going to register Victoria’s car, and Victoria is going to inflate Victoria’s tires, and Victoria is going to change Victoria’s oil.

The refund check and signed receipt must be given to the administrator in charge and the secretary at the front desk, respectively.
The refund check must be given to the administrator in charge and the signed receipt must be given to the secretary at the front desk.

You can fool some of the people all of the time, and you can fool all of the people some of the time, but you can’t fool all of the people all of the time.
You can fool some of the people all of the time, and you can fool all of the people some of the time, and you can’t fool all of the people all of the time.

Larry, Moe, and Curly are stooges.
Larry is a stooge, and Moe is a stooge, and Curly is a stooge.

Larry, Moe, and Curly are The Three Stooges.
Wrong: Larry is The Three Stooges, and Moe is The Three Stooges, and Curly is The Three Stooges.
Right: Larry, Moe, and Curly are The Three Stooges.
Right: Larry is one of The Three Stooges, and Moe is one of The Three Stooges, and Curly is one of The Three Stooges.

Julie visited an ancient and expansive home yesterday.
Julie visited an ancient and expansive home yesterday.

Julie visited an ancient and an expansive home yesterday.
Julie visited an ancient home yesterday, and Julie visited an expansive home yesterday.

These are conjunctions:
Seth stayed home, but Liz went out.
Seth stayed home, yet Liz went out.
Seth stayed home; however, Liz went out.
Seth stayed home, even though Liz went out.

This is not a conjunction:
Seth stayed home because Liz went out.

The difference is this:
If “Seth stayed home,” is true and that “Liz went out,” is true, then it is certain that “Seth stayed home, but Liz went out,” is also true. However, we cannot tell just from the fact that “Seth stayed home,” is true and that “Liz went out,” is true, whether “Seth stayed home *because* Liz went out,” is true.

3.1. Truth-functional Connectives. For connectives like ‘but’, ‘yet’, ‘and’, and ‘even though’, knowing the truth and falsity of the connected clauses is enough to tell us what the truth value of the conjunction is.

Seth stayed home. &Liz went out. &Seth stayed home, but Liz went out.
T &T &T
T &F &F
F &T &F
F &F &F

3.2. Non-truth-func-tional Connectives. For connectives like ‘because’, knowing the truth and falsity of the connected clauses is NOT enough to tell us what the truth value of the conjunction is.

Seth stayed home. &Liz went out. &Seth stayed home because Liz went out.
T &T &?
T &F &F
F &T &F
F &F &F

3.3. ‘But’ vs. ‘And’. There is no logical difference between ‘but’ and ‘and’: The truth table for them is the same. The difference in their meaning is in the conversational implications they typically possess. ‘But’ usually implies some kind of contrast from what came before.

“Seth stayed home, but Liz went out,” implies the following:

1. Seth stayed home.
2. Liz went out.
3. Liz’s going out is remarkable or surprising given that Seth’s stayed home.

Inference 1 and 2 are logical inferences because

1. You can’t consistently say “Seth stayed home, but Liz went out, and Seth didn’t stay home.”
2. You can’t consistently say “Seth stayed home, but Liz went out, and Liz didn’t go out.”
3. But, you can consistently say “Seth stayed home, but Liz went out, and it is not at all surprising or remarkable that Liz went out.”

4. Negation

Negation is the expression of a negative. Negatives appear commonly with the word ‘not’ and with negative prefixes like ‘in-’ and ‘un-’. To translate negative sentences, we first rewrite them so that they start off with the expression “It is not the case that”.

Marvin is not sick.
It is not the case that Marvin is sick.

Dexter was indispensable to his team.
It is not the case that Dexter was dispensable to Dexter’s team.

This nail has never been struck with a hammer.
It is not the case that this nail has (ever) been struck with a hammer.
Let $H =$ "This nail has been struck with a hammer."
$\neg H$

5. Subtleties with Negation

Three common cases where 'not' cannot be pulled to the front:
Intensional Contexts (belief, speech, desire)
Quantifiers (all, some, none)
Possibility/Necessity (must, might, may, should, could)

Cases where 'un-', 'in-', 'im-' cannot be pulled to the front: Contraries vs. Negations

5.1. Intensional Contexts.

In some cases, the sentence can be negative in a way that doesn't allow you to rephrase it using "It is not the case that."

Wendy said that she did not have herpes.
Wrong: It is not the case that Wendy said she did have herpes.
Right: Wendy said that she did not have herpes.

It is written on that sign that smoking is not permitted.
Wrong: It is not the case that it is written on that sign that smoking is permitted.
Right: It is written on that sign that smoking is not permitted.

Negatives that occur in the context of speech, writing, belief, thought, conjecture, etc. cannot usually be translated as a negation. These contexts are called intensional contexts.

Whenever you have
'believes that',
'thinks that',
'said that',
'wrote that',
'wished that',
you often won't be able to extract the logic from what follows. If the rephrased sentence doesn't mean the same thing as the original, you should just treat the entire belief statement (or speech statement) as a simple sentence.

Usually, you will be able to extract conjunctions, but usually not negations, and usually not disjunctions.

Good Translation:
The official thought that the Kenyan and the Norwegian had finished the race.
The official thought that the Kenyan had finished the race, and the official thought that the Norwegian had finished the race.

Bad Translation:
Tim instructed his mother not to go into his room.
It is not the case that Tim instructed his mother to go into his room.

Questionable Translation:
The kid wants either cherry or strawberry. (He just wants one of the red flavors, and he doesn't care which.)
Either the kid wants cherry or the kid wants strawberry. (He wanted a specific flavor, but I forget which one. All I can remember is that it is a red flavor.) More Examples

Good Translation:
He didn't believe that he wasn't going to make it home.
It is not the case that he believed that he wasn't going to make it home.

Good Translation:
Alice didn't say she was in trouble.
It is not the case that Alice said she was in trouble.

Is the next one a good translation?
I don't know whether we'll have a blizzard this week.
It is not the case that I know whether we'll have a blizzard this week.

Is the next one a good translation?
I don't think Jill is creative.
It is not the case that I think Jill is creative.

Remember: Statements are logically equivalent if they are true in all the same circumstances (and false in all the same circumstances).

5.2. Quantifiers. Whenever you have words like
'everyone',
'all',
'someone',
'some',
you often won't be able to extract the logic from what follows. If the rephrased sentence doesn't mean the same thing as the original, you should just treat the entire statement as a simple sentence.

Usually, you will be able to extract conjunctions, but usually not negations, and usually not disjunctions.

Good Translation:
Everyone is wearing a cheap tie-dye T-shirt and beret.
Everyone is wearing a cheap tie-dye T-shirt, and everyone is wearing a beret.

Bad Translation:
Some people don't take a shower everyday.
It is not the case that some people take showers everyday.

Bad Translation:
Everyone is either male or female.
Either everyone is male or everyone is female.

5.3. Negative Quantifiers. 'no one' = 'not someone' = 'not one person' = 'not one or more people'
'nobody' = 'not somebody' = 'not one person' = 'not one or more people'
'nothing' = 'not something' = 'not one thing' = 'not one or more things'
'nowhere' = 'not somewhere' = 'not one place' = 'not one or more places'

No one caught you smoking dope.
It is not the case that someone caught you smoking dope.

Nobody wants to be your friend.
It is not the case that somebody wants to be your friend.

Nothing is forgiven in this household.
It is not the case that something is forgiven in this household.
It is not the case that one or more things are forgiven in this household.

The contraband is nowhere to be found.
It is not the case that the contraband is somewhere to be found.

5.4. Possibility/Necessity. Whenever you have words like
'might',
'must',
'may',
'possible',
'necessary',
'need',
'should',
'ought',
you often won't be able to extract the logic from what follows. If the rephrased sentence doesn't mean the same thing as the original, you should just treat the statement as a simple sentence.

Good Translation:
Cliff must own a bike and a helmet.
Cliff must own a bike, and Cliff must own a helmet.

Bad Translation:
Cliff might own a bike and a helmet.
Cliff might own a bike, and Cliff might own a helmet.

Bad Translation:
Amy must not go to Vegas.
It is not the case that Amy must go to Vegas.

Bad Translation:
It's possible that you don't have chlamydia.
It is not the case that it is possible that you have chlamydia.

5.5. Contraries vs. Negations. Example of contraries: red and blue
Example of negations: colored and colorless

Which translations are good?

$$\frac{\textit{Joann is unhappy.}}{\textit{It is not the case that Joann is happy.}}$$

$$\frac{\textit{Reggie was unfazed by the insult.}}{\textit{It is not the case that Reggie was fazed by the insult.}}$$

$$\frac{\textit{We stumbled upon some untoward consequences.}}{\textit{It is not the case that we stumbled upon some toward consequences.}}$$

$$\frac{\textit{Antonio's shoelace is unraveled.}}{\textit{It is not the case that Antonio's shoelace is raveled.}}$$

Cindy didn't invite Nathan, and we didn't press the issue with her.
Right: It is not the case that Cindy invited Nathan, and it is not the case that we pressed the issue with Cindy.
N = "Cindy invited Nathan."
I = "We pressed the issue with Cindy."
$\neg N \& \neg I$

5.6. Negation and Conjunction Together. Chameleons never eat birds, but only insects and fruit.
Wrong: It is not the case that chameleons eat birds, and chameleons only eat insects and chameleons only eat fruit.
Right: It is not the case that chameleons eat birds, and chameleons only eat insects

and fruit. B = "Chameleons eat birds."
E = "Chameleons only eat insects and fruit."
$\neg B \& E$

Both Bill and Ted did not respond.
Wrong: It is not the case that (Bill responded and Ted responded).
Right: It is not the case that Bill responded, and it is not the case that Ted responded.
B = "Bill responded."
T = "Ted responded."
$\neg B \& \neg T$

Bill and Ted did both not respond.
Wrong: It is not the case that Bill responded, and it is not the case that Ted responded.
Right: It is not the case that both (Bill responded and Ted responded).
B = "Bill responded."
T = "Ted responded."
$\neg(B \& T)$

Summary of technique:
Step 1: Try to pull the negations (not, un-, im-) out in front of the sentence by making them "It is not the case that"
Step 2a: Hard but always works in theory:
Check the meaning of the two sentences. Are they true in all the same circumstances?
Step 2b: Easy but doesn't always work.
Make sure you don't pull the negation through words that express modality (might, must), intentionality (thought, believed), quotation (said, wrote), or quantity (all, some, none).
Step 3: Anywhere you see "It is not the case that" you put a ¬. Anywhere you see an 'and' connecting sentences, put a '&'. Anywhere you see words that match a given atomic sentence, put the corresponding letter down.

6. Scope

It is not the case that Trevor left his book behind and Trevor went to class.
B = "Trevor left Trevor's book behind."
C = "Trevor went to class."

Did Trevor go to class? Unclear
You could translate it in two different ways.
$\neg B \& C$
$\neg(B \& C)$

6.1. Well-formed Formulas. In our formal language, we don't want any ambiguity, so we have to make rules that leave the structure of the sentence perfectly clear. We do this by setting up some formal rules for what counts as a well-formed formula (wff):

Any capital letter is a wff.
If α is a wff, then $\neg\alpha$ is a wff.
If α is a wff, and β is a wff, then $(\alpha \& \beta)$ is a wff.
If α is a wff, and β is a wff, then $(\alpha \vee \beta)$ is a wff.
If α is a wff, and β is a wff, then $(\alpha \supset \beta)$ is a wff.
Nothing else is a wff.
(As a relaxing of the rules, we allow ourselves to drop the outermost parentheses.

Why have this new concept of the wff? (Why not just talk about what statements are legal?) There's no good reason yet. In propositional logic, a wff is the same thing as a statement, but later, with more complicated logic, they will be different. We are just getting used to the terminology that we will have to use later.

Which of the following are legitimate sentences (wffs)?

Example 4.

$$(\neg X \& \neg X)$$
$$(P \& Q \& R)$$
$$\neg(E \vee g)$$
$$(\neg T \supset (T \vee \neg\neg\neg S))$$
$$((A \& B) \& (\neg Y \& R)$$
$$(\vee R \& P)$$
$$(F \neg F)$$
$$(T \& H \neg)$$
$$((I \neg \& W) \supset K)$$
$$\neg\neg P \& Q$$

The scope of a connective is the expression containing the connective, any the wffs it connects, any parentheses that directly enclose the connective, and nothing else.

Examples from Arithmetic:

- The scope of the '$\times$' in the expression $4 + (3 \times 5)$ is marked in bold blue: $4 + \mathbf{(3 \times 5)}$.

- The scope of the '+' in the expression $4 + (3 \times 5)$ is marked in bold blue: $\mathbf{4 + (3 \times 5)}$.
- The scope of the '-' in the expression $-8 + (1 + 2)$ is marked in bold blue: $\mathbf{-8} + (1 + 2)$.
- The scope of the '-' in the expression $6 + -(1 + 2)$ is marked in bold blue: $6 + \mathbf{-(1 + 2)}$.

The scope of an arithmetic operator is the operator, the numbers it connects and any parentheses around them.

Scope in propositional logic works much like in arithmetic, but a little easier. To make legitimate wffs, we have to put parentheses around any use of &,∨, or ⊃. So you can think of the &, ∨, and ⊃ like '+' signs and the ¬ like the minus sign that indicates a negative number.

The scope of the second & is marked in bold blue: $(K\&\neg\mathbf{(B\&H)})$.
The scope of the ¬ is marked in bold blue: $(K\&\mathbf{\neg(B\&H)})$.
The scope of the first & is marked in bold blue: $\mathbf{(K\&\neg(B\&H))}$.

Remember: $(A\neg B)$ is not a legitimate sentence. The '¬' does not work like a minus sign that signifies subtraction. In arithmetic, (5-3) is shorthand for (5 + -3). In logic, we have to write out sentences like $(A\&\neg B)$. No shorthand. We use a bit of shorthand by optionally omitting the outermost parentheses. But the rules otherwise stay the same.

The scope of the ∨ is marked in bold blue: $\neg P\&\mathbf{(Q \vee \neg P)}$. The scope of the first ¬ is marked in bold blue: $\mathbf{\neg P}\&(Q \vee \neg P)$. The scope of the second ¬ is marked in bold blue: $\neg P\&(Q \vee \mathbf{\neg P})$. The scope of the & is marked in bold blue: $\mathbf{\neg P\&(Q \vee \neg P)}$.

$$(A\&B)\&\neg(C\&D)$$
$$\neg(A\&B)\&(C\&D)$$
$$\neg((A\&B)\&(C\&D))$$
$$\neg\neg(A\&B)\&\neg(C\&D)$$

The **main connective** is the connective whose scope is the entire wff.

Examples:
The main connective in $(R \vee \neg\neg E)\&\neg(P \supset W)$ is the &.
The main connective in $\neg((\neg G \vee T) \supset \neg Q)$ is the first ¬.

What is the main connective in the sentences below?

$$\neg(S \vee \neg Q)\&P$$
$$L\&\neg(R \supset \neg\neg E)$$
$$\neg(R \vee \neg W) \supset \neg(P\&\neg R)$$
$$\neg(Y \supset \neg(W\&\neg E))$$

Helpful rule: If the outermost parentheses are missing (as they usually are), look at connectives that are outside all parentheses. If there's a connective that is not a negation, that's the main connective. Otherwise it is the negation.

7. Truth Table for Conjunction

We define the meaning of '&' by making it match up with our intuitive understanding of the word 'and':

Seth stayed home. &Liz went out. &Seth stayed home, and Liz went out.
T &T &T
T &F &F
F &T &F
F &F &F

A &B &$A\&B$
T &T &T
T &F &F
F &T &F
F &F &F

Table 1. Truth Table for Conjunction

8. Truth Table for Negation

We define the meaning of '¬' by making it match up with our intuitive understanding of the expression 'it is not the case that':

Barney has a big nose. &It is not the case that Barney has a big nose.
T &F
F &T

B &$\neg B$
T &F
F &T

Table 2. Truth Table for Negation

9. Truth Tables

Step 1: Label one column for each sentence letter in the statement and a column for the statement whose truth one wants to evaluate.

Step 2: For n sentence letters, make 2^n rows. (For 1 letter make 2 rows; for 2 letters make 4 rows; for 3 letters make 8 rows; for 4 letters make 16 rows; for 5 letters make 32 rows; etc.)

Step 3: Put T's and F's in the columns for the sentence letters. For the first column, alternate T and F. For the second column, alternate T T then F F. The last column should always have the top half T and the bottom half F. If not, you made a mistake or you didn't make the right number of rows.

Optional Step 4: Fill in T's and F's underneath all instances of the sentence letters, making it consistent with the columns on the left. For example, under the C there should be a pattern of T's and F's matching the column on the left marked C.

Step 5: Fill in T's and F's under the sentence connectives like & and ¬ whenever you have enough information to do so. Use the rules for these connectives (their truth tables) to let you know what letter to put under them. For example, the magenta letter is a T because $(A\&C)$ is true when A is true and C is true. The

A &*B* &*C* &(*A*&*C*) && &¬*B*
T &T &T &&&
T &T &F &&&
T &F &T &&&
T &F &F &&&
F &T &T &&&
F &T &F &&&
F &F &T &&&
F &F &F &&&

A &*B* &*C* &(*A*&*C*) && &¬*B*
T &T &T &T T &F &F
T &T &F &T F &F &T
T &F &T &T T &F &F
T &F &F &T F &F &T
F &T &T &F T &F &F
F &T &F &F F &F &T
F &F &T &F T &F &F
F &F &F &F F &F &T

blue letter is an F because the conjunction (*A*&*C*) is false when *A* is false and *C* is true.

A &*B* &*C* &(*A*&*C*) && &¬*B*
T &T &T &TTT &F &FT
T &T &F &TFF &F &TF
T &F &T &TTT &F &FT
T &F &F &TFF &F &TF
F &T &T &FFT &F &FT
F &T &F &FFF &F &TF
F &F &T &FFT &F &FT
F &F &F &FFF &F &TF

To figure out what letter goes under the &, you need to look at the what the & connects: (*A*&*C*) and ¬*B*. Then look under the column of the main connective for each of them. The main connective of (*A*&*C*) is & and the main connective of ¬*B* is ¬. So we take the T under the & and the F under the ¬ and we use the truth table for conjunction to tell us that a T and an F conjoined together gives F for an answer. Thus, the green letter is F.

Step 6: After every connective has a full column of T's and F's, circle the connective with the widest scope. This is column is identified here by being in boldface blue.

Truth tables tell us what statements are tautologies, what statements are contradictions and what statements are contingent.

We look at the boldface blue column.

- If we see all T's, the statement is a tautology.
- If we see all F's, the statement is a contradiction.
- If we see some T's and some F's, the statement is contingent.

A &*B* &*C* &(*A*&*C*) && &¬*B*
T &T &T &TTT &**F** &FT
T &T &F &TFF &**F** &TF
T &F &T &TTT &**F** &FT
T &F &F &TFF &**F** &TF
F &T &T &FFT &**F** &FT
F &T &F &FFF &**F** &TF
F &F &T &FFT &**F** &FT
F &F &F &FFF &**F** &TF

EXERCISE 6. *Prove that the following is a contradiction: "Lithium chlorate is an acid, and it's not an acid."*
First, we translate it into "Lithium chlorate is an acid, and it's not the case that lithium chlorate is an acid."
Let A *= "Lithium chlorate is an acid."*
Then, we translate into symbols: (A&¬A)
Then, we do the truth table. The column under the main connective is all F's, so

A &*A*&¬*A*
T &*T***F***FT*
F &*F***F***TF*

it's a contradiction.

EXERCISE 7. *Prove that the following statements are equivalent:*
"Keith went to the movies, and Chuck did too."
"Chuck went to the movies, and Keith did as well."
First, we translate them into
"Keith went to the movies, and Chuck went to the movies."
"Chuck went to the movies, and Keith went to the movies."
Let K *= "Keith went to the movies."*
Let C *= "Chuck went to the movies."*
Then we translate into symbols: (K&C) *and* (C&K).
Then, we do the truth table. Because both columns under the main connective are

K &*C* &*K*&*C* &*C*&*K*
T &*T* &*T***T***T* &*T***T***T*
F &*T* &*F***F***T* &*T***F***F*
T &*F* &*T***F***F* &*F***F***T*
F &*F* &*F***F***F* &*F***F***F*

the same in every row, they are equivalent.

EXERCISE 8. *Prove that the following statements are equivalent:*
"It's untrue that Amy didn't pass out."
"Amy passed out."
First, we translate into
"It's not the case that it's not the case that Amy passed out."
"Amy passed out."

Let A = "Amy passed out."

Then we translate into symbols:

$\neg\neg A$

A

Then, we do the truth table. Because both columns under the main connective

$A\ \&\neg\neg A\ \&A$
$T\ \&\mathbf{T}FT\ \&\mathbf{T}$
$F\ \&\mathbf{F}TF\ \&\mathbf{F}$

are the same in every row, they are equivalent.

Exercise 9. *Problem: Show that the two ways of translating "Inky, Blinky, and Pinky are ghosts," are equivalent.*

Let I = "Inky is a ghost."

Let B = "Blinky is a ghost."

Let P = "Pinky is a ghost."

We can translate it as $((I\&B)\&P)$ *or as* $(I\&(B\&P))$. *This means we don't*

$I\ \&B\ \&P\ \&((I\&B)\&P)\ \&(I\&(B\&P))$
$T\ \&T\ \&T\ \&TTT\mathbf{T}T\ \&T\mathbf{T}TTT$
$F\ \&T\ \&T\ \&FFT\mathbf{F}T\ \&F\mathbf{F}TTT$
$T\ \&F\ \&T\ \&TFF\mathbf{F}T\ \&T\mathbf{F}FFT$
$F\ \&F\ \&T\ \&FFF\mathbf{F}T\ \&F\mathbf{F}FFT$
$T\ \&T\ \&F\ \&TTT\mathbf{F}F\ \&T\mathbf{F}TFF$
$F\ \&T\ \&F\ \&FFT\mathbf{F}F\ \&F\mathbf{F}TFF$
$T\ \&F\ \&F\ \&TFF\mathbf{F}F\ \&T\mathbf{F}FFF$
$F\ \&F\ \&F\ \&FFF\mathbf{F}F\ \&F\mathbf{F}FFF$

have to worry about how to place the parentheses in translating sentences with 2 conjunctions. They are logically equivalent.

Exercise 10. *Show that the two sentences below mean different things:*

"Heather and Stephanie did not both go to the bar."

"Heather and Stephanie both did not go to the bar."

H = "Heather went to the bar."

S = "Stephanie went to the bar."

$\neg(H\&S)$

$(\neg H\&\neg S)$

$H\ \&S\ \&\neg(H\&S)\ \&\neg H\&\neg S$
$T\ \&T\ \&\mathbf{F}TTT\ \&FT\mathbf{F}FT$
$F\ \&T\ \&\mathbf{T}FFT\ \&FT\mathbf{F}TF$
$T\ \&F\ \&\mathbf{T}TFF\ \&TF\mathbf{F}FT$
$F\ \&F\ \&\mathbf{T}FFF\ \&TF\mathbf{T}TF$

They have two rows (the blue ones) where the sentences differ in truth value. Thus, they are inequivalent.

Exercise 11. *Evaluate the following argument for validity.*

$\neg(A\&\neg B)$
$\underline{\neg(B\&A)}$
$(A\&\neg B)$

$A \;\&B \;\&\neg(A\&\neg B) \;\&\neg(B\&A) \;\&\; (A\&\neg B)$
T &T &**T**TFFT &**F**TTT &T**F**FT
F &T &**T**FFFT &**T**TFF &F**F**FT
T &F &**F**TTTF &**T**FFT &T**T**TF
F &F &**T**FFTF &**T**FFF &F**F**TF

The truth table shows (in blue) that we have a counterexample when A is false and B is true and we have another counterexample when A and B are both false. Thus, the argument is invalid.

Example argument:

It turned out to be untrue that Eve, Regina and Donna all did their own homework.
Jason falsely claimed, "Donna did her own homework, but Regina did someone else's instead."
Donna did do her own homework.
Thus, Eve didn't do her homework.

We translate this into:

It is not the case that (Eve did Eve's homework and Regina did Regina's homework and Donna did Donna's homework).
It is not the case that (Donna did Donna's homework, and it is not the case that Regina did Regina's homework).
Donna did Donna's homework.
Thus, it is not the case that Eve did Eve's homework.

Let D = "Donna did Donna's homework."
Let R = "Regina did Regina's homework."
Let E = "Eve did Eve's homework."

$\neg((E\&R)\&D)$
$\neg(D\&\neg R)$
$\underline{D}$
$\neg E$

$E \;\&D \;\&R \;\&\neg((E\&R)\&D) \;\&\neg(D\&\neg R) \;\&D \;\&\neg E$
T &T &T &**F**TTTTTT &**T**TFF &**T** &**F**T
F &T &T &**T**FFTFT &**T**TFF &**T** &**T**F
T &F &T &**T**TTTFF &**T**FFF &**F** &**F**T
F &F &T &**T**FFTFF &**T**FFF &**F** &**T**F
T &T &F &**T**TTFFT &**F**TTT &**T** &**F**T
F &T &F &**T**FFFFT &**F**TTT &**T** &**T**F
T &F &F &**T**TFFFF &**T**FFT &**F** &**F**T
F &F &F &**T**FFFFF &**T**FFT &**F** &**T**F

The proof is in the truth table. There are no counterexamples. Thus, the argument is valid.

10. Translating Negations with Conjunctions

11. Disjunction

Disjunction is the fancy name for 'or'. It is usually expressed with 'or' or with "either or" Just like with conjunction, you should express disjunctions by breaking them up into statements connected with 'or'. Disjunction is symbolized with the wedge: '$\vee$'.

Dave has cable or a satellite dish.
Dave has cable, or Dave has a satellite dish.

Either Trent or Evan won the tournament.
Trent won the tournament or Evan won the tournament.

You should be able to understand what I'm saying, or you're an idiot.
Let U = "You should be able to understand what I am saying."
Let I = "You are an idiot."
$U \vee I$

Just as with negation you have to watch out for quantifiers, psychological verbs, and for sentences that express possibility or necessity.

Bad Translation:
Every whole number is either odd or even.
Every whole number is even, or every whole number is odd.

Questionable Translation:
People have to mark the form with either 'male' or 'female'.
People have to mark the form with 'male', or people have to mark the form with 'female'.

Questionable Translation:
The kid wants either cherry or strawberry. (He just wants one of the red flavors, and he doesn't care which.)
Either the kid wants cherry or the kid wants strawberry. (He wanted a specific flavor, but I forget which one. All I can remember is that it is a red flavor.)

11.1. Truth Table for Disjunction.

This is the truth table for $\vee$:

A &B &$A \vee B$
T &T &T
F &T &T
T &F &T
F &F &F

Table 3. Truth Table for Disjunction

The last three lines are totally uncontroversial, but the top line has at least has generated some debate over whether '$\vee$' accurately captures the logic of the English 'or'.

The inclusive sense of 'or' is the interpretation where we interpret "A or B" as "Either A or B or both." The exclusive sense of 'or' is the interpretation where we interpret "A or B" as "Either A or B, but not both.".

Here are three arguments why the English 'or' is inclusive (i.e., given by the truth table for '$\vee$'.

Argument I: Suppose you are coming back from the party where you saw many friends including Julie and Elaine. Dirk stops you outside on the street and asks you the following: "I'm looking for my roommate, and I need to find someone who has seen him lately. Have you seen either Julie or Elaine tonight?" What is the most honest response you can give, 'Yes' or 'No'? The answer is 'Yes.' That means you agree with the statement, "I have seen either Julie or Elaine tonight." That means you think the statement, "I have seen Julie tonight, or I have seen Elaine tonight," is true.

Argument II: Suppose someone says, "Either she had heart surgery, or she had brain surgery." That seems to imply she didn't have both surgeries. We can test this out by trying to cancel the inference. We consider, "Either she had heart surgery, or she had brain surgery, and I think she had both surgeries." If the inference were logical, this sentence would sound contradictory, but it doesn't. Thus, the inference is conversational, not logical.

Argument III: Suppose Jeff plans to go to the Metallica concert and to sleep there, but he says, "I'll either go to the Metallica concert tonight, or I'll catch up on some sleep." Jeff is probably being misleading because it is improper conversational etiquette to say what is less informative when you are in a position to be more informative.

It is similar to when Jeff tells his girlfriend Lisa that he is going over to Dave's house, when in fact he is planning to go over to Dave's house for a token appearance before going next door to pick up Jennifer for their date.

12. Examples using Disjunction

Problem: Show that $(A \vee \neg A)$ is a tautology.

A &$A \vee \neg A$
T &T**T**FT
F &F**T**TF

Problem: Show $(A \vee B)$ is logically equivalent to $(B \vee A)$.

K &C &K&C &C&K
T &T &T**T**T &T**T**T
F &T &F**F**T &T**F**F
T &F &T**F**F &F**F**T
F &F &F**F**F &F**F**F

Problem: Prove that the following argument is valid.

Ethan is going to both Bermuda and Oslo during his vacation.

During his vacation, Ethan is either going to Bermuda or to Olso.

B = "Ethan is going to Bermuda during his vacation."
O = "Ethan is going to Oslo during his vacation."

There are no rows where the $B\&O$ is true and the $B \vee O$ is false. Thus, the argument is valid.

Problem: Show that $((P \vee R) \vee S)$ and $(P \vee (R \vee S))$ are logically equivalent.

B &O &$B\&O$ &$B \vee O$
T &T &T**T**T &T**T**T
F &T &F**F**T &T**T**F
T &F &T**F**F &F**T**T
F &F &F**F**F &F**F**F

P &R &S &$((P \vee R) \vee S)$ &$(P \vee (R \vee S))$
T &T &T &TTT**T**T &T**T**TTT
F &T &T &FTT**T**T &F**T**TTT
T &F &T &TTF**T**T &T**T**FTT
F &F &T &FFF**T**T &F**T**FTT
T &T &F &TTT**T**F &T**T**TTF
F &T &F &FTT**T**F &F**T**TTF
T &F &F &TTF**T**F &T**T**FFF
F &F &F &FFF**F**F &F**F**FFF

P &R &S &$((P\&R) \vee S)$ &$(P\&(R \vee S))$
T &T &T &TTT**T**T &T**T**TTT
F &T &T &FFT**T**T &F**F**TTT
T &F &T &TFF**T**T &T**T**FTT
F &F &T &FFF**T**T &F**F**FTT
T &T &F &TTT**T**F &T**T**TTF
F &T &F &FFT**F**F &F**F**TTF
T &F &F &TFF**F**F &T**F**FFF
F &F &F &FFF**F**F &F**F**FFF

Problem: Show that $((P\&R) \vee S)$ and $(P\&(R \vee S))$ are logically inequivalent.
Problem: Show that $\neg(X\&Y)$ and $(\neg X \vee \neg Y)$ are logically equivalent.

X &Y &$\neg(X\&Y)$ &$(\neg X \vee \neg Y)$
T &T &**F**TTT &FT**F**FT
F &T &**T**FFT &TF**T**FT
T &F &**T**TFF &FT**T**TF
F &F &**T**FFF &TF**T**TF

Problem: Show that $\neg(X \vee Y)$ and $(\neg X\&\neg Y)$ are logically equivalent.

12.1. Translating Disjunctions. If the ‘not’ (possibly from the ‘n’ in ‘neither’) is on the left of the ‘either’, put the ‘¬’ on the left of the disjunction. $\neg(D \vee C)$ If the ‘not’ is on the right of the ‘either’, make a disjunction of negations, $(\neg D \vee \neg C)$, or a disjunction with only one ‘˜’ if there is an extra verb to make clear where the disjunction should not go.

Either Dave or Carol is going. Neither Dave nor Carol are going. Either Dave or Carol is not going. Either Dave is or Carol is not going.

Kelly and Margaret or Rex and Jeff are going to the movies.

Kelly and Margaret or Rex are going to the movies. Ambiguous, but probably Kelly and either Margaret or Rex are going to the movies.

Either Kelly or Margaret and Rex are going to the movies. Still ambiguous, but probably

X &Y &$\neg(X \vee Y)$ &$(\neg X \& \neg Y)$
T &T &**F**TTT &FT**F**FT
F &T &**F**FTT &TF**F**FT
T &F &**F**TTF &FT**F**TF
F &F &**T**FFF &TF**T**TF

Christine or Shannon is not going to be in the play.

Either Christine and Shannon are not or Taylor is going to be in the play.

Either Christine or Shannon are not and Taylor is going to be in the play.

Neither Christine nor Shannon are not going to be in the play.

Neither Christine nor Shannon nor Taylor are going to be in the play.

Notice how there are two devices for disambiguating the scope of sentences that involve disjunctions and conjunctions. The first is the use of shortening the conjunction or disjunction of independent clauses into a conjunction or disjunction of the subject.

Let B = "Bill drove the bus."
Let T = "Ted drove the bus."
Let W = "We waited."

"Either Bill and Ted drove the bus, or we waited.": $(B\&T) \vee W$

"Either Bill or Ted drove the bus, and we waited.": $(B \vee T)\&W$

The second device is to use the "Either...or..." to bracket what counts as the disjunct.

"Either Bill drove the bus, or Ted drove, and we waited.": $B \vee (T\&W)$

"Either Bill drove the bus, and we waited, or Ted drove.": $(B\&W) \vee T$

Let M = "Melanie slept."
Let K = "Kara slept."
Let J = "Jill stayed up."

"Jill stayed up, and Kara slept, or Melanie slept." (Ambiguous)
"Jill stayed up and Kara or Melanie slept.": $J\&(K \vee M)$
"Jill stayed up and either Kara slept or Melanie slept.": $J\&(K \vee M)$

"Either Jill stayed up and Kara slept or Melanie slept.": $(J\&K) \vee M$

The "neither... nor..." construction is easy to translate. It means "not-(either... or...)." "Neither Melanie nor Kara slept."
"It is not the case that [either Melanie or Kara slept]."
$\neg(M \vee K)$ which is equivalent to $\neg M\&\neg K$.

'Neither' and 'nor' sometimes exists without each other in which case they mean 'also not.'
"Kara didn't stay up, and neither did Melanie.": $\neg K\&\neg M$.
"Kara didn't stay up, nor did Melanie.": $\neg K\&\neg M$.

There are some times when the English 'or' is really used as a conjunction. Here is one example:
"Ms. Thompson offered us tea or coffee."
"Ms. Thompson offered us tea, and Ms. Thompson offered us coffee."
Let T = "Ms. Thompson offered us tea."
Let C = "Ms. Thompson offered us coffee."
$T\&C$

13. Conditionals

As we know from our rules, $(A \supset C)$ is a sentence.

Some terminology: The sentence before the '⊃' is called the antecedent. The sentence after the '⊃' is called the consequent.

antecedent ⊃ consequent

The truth table for '⊃' is

A &C &$A \supset C$
T &T &T
F &T &T
T &F &F
F &F &T

TABLE 4. Truth Table for the Material Conditional

The conditional is true if either the antecedent is false or the consequent is true. Otherwise, it is false. The truth table for '⊃' is all we need to know to calculate complex expressions using it.

Here is the truth table for $\neg(P \supset S) \supset (R\&S)$

P &R &S &$\neg(P \supset S) \supset (R\&S)$
T &T &T &**F**TTTTTTT
F &T &T &**F**TTTTTTT
T &F &T &**F**TTTTFFT
F &F &T &**F**FTTTFTT
T &T &F &**T**TFFFTFF
F &T &F &**F**FTFTTFF
T &F &F &**T**TFFFFFF
F &F &F &**F**FTFTFFF

13.1. Translations to and from English. Consider $J \supset C$ with
J = "Jason bought a new skateboard."
C = "Cliff will be impressed."

Here are different idioms we have in English to express this idea:
"If Jason bought a new skateboard, then Cliff will be impressed."
"Cliff will be impressed if Jason bought a new skateboard."

There are some other expressions that also get translated with '⊃':

> "The match lit only if there was oxygen in the room."
> L = "The match lit."
> O = "There was oxygen in the room."
> $L \supset O$
>
> "We could hear the cannon blast only if the cannon was fired."
> H = "We could hear the cannon blast ."
> F = "The cannon was fired."
> $H \supset F$
>
> "The only way the couch could have caught on fire is if you were smoking in my apartment."

"The couch was able to catch on fire only if you were smoking in my apartment."
C = "The couch was able to catch on fire."
S = "You were smoking in my apartment."
$C \supset S$

'$\supset$' means 'only if'. However, if you try to translate the '$\supset$' as 'only if' that will sometimes lead to funny translations:
K = "Kelly flipped the switch."
L = "The light came on."

How would we translate $K \supset L$?:
"Kelly flipped the switch only if the light came on."

Awkward but logically correct:
"Kelly flipped the switch only if the light came on."

More natural:
"If Kelly flipped the switch, then the light came on."

13.2. Unless. Sentences with 'unless' make a claim, and then give an escape clause. If the escape condition is not satisfied, then the claim stands as stated. If the escape condition is satisfied, the claim is effectively retracted.

Suppose I say: "Unless my car is broken, I will pick you up."
My claim is a promise that I will pick you up. However, I added an escape clause so that if my car is broken, I'm no longer obligated to pick you up.
Case I: My car is not broken. In this case, I am obligated to pick you up.
Case II: My car is broken. In this case, I might pick you up and I might not. It depends on whether I can get alternative transportation. I'm not making any promises one way or the other.

"Unless my car is broken, I will pick you up." means the same thing as "If my car is not broken, I will pick you up." So we translate it as follows:

C = Claim = "I will pick you up."
E = Escape clause = "My car is broken."
$\neg E \supset C$

How do we translate "We are going to crash unless you get out of the way"?

C = "We are going to crash."
E = "You get out of the way."
$\neg E \supset C$

13.3. If and only if. This only gets used in philosophy and mathematics. It has the obvious meaning. A if and only if B means A if B, and A only if B which means (If B, then A) and (A only if B) which means $(B \supset A)\&(A \supset B)$. Example: "A polygon is a triangle if and only if it has three sides."

13.4. Summary of Translation Rules.

Let G = "Gina travels."
Let H = "Jill travels."

When a sentence includes 'if,' first check whether it appears as part of an 'only if' or an 'even if' or an 'if and only if.' If it does, use the special rules for these idioms:

- "Gina travels only if Jill does," becomes $G \supset J$.

- "Only if Jill travels, does Gina travel," becomes $G \supset J$.
- "Gina travels even if Jill does," becomes G.
- "Even if Jill travels, Gina travels," becomes G.
- "Gina travels if and only if Jill does," becomes $(G \supset J)\&(J \supset G)$.

In cases where the 'if' is not part of such an idiom, spell out the sentence in 'if...then...' form.

- "Jill travels, if Gina does," becomes $G \supset J$.
- "If Gina travels, then Jill does," becomes $G \supset J$.
- "If Gina travels, so does Jill," becomes $G \supset J$.

The cases where conjunctions are involved have their scope disambiguated by looking at where the 'if' and 'then' are located. Anything between the 'if' and the 'then' is bracketed together on the left side of the '$\supset$' and everything else (up to the end of the conditional) is bracketed on the right side. Also if subjects or direct objects are conjoined, they are not split by the '$\supset$'.
Let K = "Kelly travels."

- "If Gina and Kelly travel, then Jill does," becomes $(G\&K) \supset J$.
- "If Gina travels, then Kelly and Jill do," becomes $G \supset (K\&J)$.
- "*If* Gina travels, and Kelly travels, *then* Jill travels," becomes $(G\&K) \supset J$.
- "*If* Gina travels, *then* if Kelly travels, Jill travels," becomes $G \supset (K \supset J)$.
- "*If* Gina travels if Kelly does, *then* Jill travels," becomes $(G \supset K) \supset J$.

With 'unless', we call the clause that immediately follows the 'unless' and call it the escape clause. The other clause is called the main clause. Then we put the negation of the escape clause on the left side of '$\supset$' and the main clause on the right.

- "Unless Gina travels, Jill does," becomes $\neg G \supset J$.
- "Unless Gina doesn't travel, Jill does," becomes $\neg\neg G \supset J$.
- "Jill travels unless Gina does," becomes $\neg G \supset J$.
- "Unless Gina and Kelly travel, Jill does," becomes $\neg(G\&K) \supset J$.
- "Unless Gina travels, Kelly and Jill travel," becomes $\neg G \supset (K\&J)$.
- "Unless Gina and Kelly don't travel, Jill does," becomes $\neg\neg(G\&K) \supset J$.

13.5. The English Conditional vs. The Material Conditional. The English conditional is the name for sentences of English of the form "If α, then β," or "α only if β," or "α unless β," or some similar expression.

The material conditional is the name for sentences of logic of the form '$\alpha \supset \beta$'.

The philosophical question we want to answer is, "How good of a job does our logic do of translating English conditionals?"

The English conditional makes a claim that is only in force under certain specified circumstances (i.e. when the antecedent is true).

Suppose I say, "If the sign is octagonal, then it's red." What I am doing is conditionally asserting that the sign is red. There are two ways things can turn out:

(1) The sign is octagonal. In this case, I am committed to the statement that the sign is red.
(2) The sign is not octagonal. In this case, I am not committed to anything.

It's octagonal. &It's red. &If it's an octagon, then it's red.
T &T &T
F &T &F
T &F &?
F &F &?

Two reasons for thinking that '$\supset$' does a good job of capturing the meaning of English conditionals: 1. Conditionals that are promises have the same truth table as '$\supset$'. 2. Conditionals can be translated as a disjunction.

Argument I: Consider "If you get hurt, I will help you."
Think about the conditional as a promise:
If I promise you something and I keep the promise, then my promise is true.
If I promise you something and I break the promise, then my promise is false.

You get hurt. &I will help you. &If you get hurt, I will help you.
T &T &T (I kept my promise)
F &T &T (I kept my promise)
T &F &F (I broke my promise)
F &F &T (I kept my promise)

Argument II: Consider the following translations:
"If I left my keys at home, then my roommate can bring them."
"Either I did not leave my keys at home, or I did leave my keys at home and my roommate can bring them."

"If you ate the pizza, then the fridge is empty."
"Either you didn't eat the pizza, or you did eat the pizza and the fridge is empty."
P = "You ate the pizza."
E = "The fridge is empty."

$P \,\&E \,\&\neg P \vee (P\&E) \,\&P \supset E$
T &T &FT**T**TTT &T**T**T
F &T &TF**T**FFT &F**T**T
T &F &FT**F**TFF &T**F**F
F &F &TF**T**FFF &F**T**F

13.6. Examples of Arguments. Consider the following argument:

If you mow my lawn, I will pay you \$20.
If you don't my lawn, I won't pay you \$20.

M = "You mow my lawn."
P = "I will pay you \$20."

$$\frac{M \supset P}{\neg M \supset \neg P}$$

Is this valid? No

Consider the following argument in the context of a bet about US history:
If James Madison was a US president, then I will pay you \$20.
If I don't pay you \$20, then James Madison was not a US president.

M &P &$M \supset P$ &$\neg M \supset \neg P$
T &T &T**T**T &FT**T**FT
F &T &F**T**T &TF**F**FT
T &F &T**F**F &FT**T**TF
F &F &F**T**F &TF**T**TF

M = "James Madison was a US president."
P = "I will pay you $20."

$M \supset P$
$\neg P \supset \neg M$

Is this valid? Yes

M &P &$M \supset P$ &$\neg P \supset \neg M$
T &T &T**T**T &FT**T**FT
F &T &F**T**T &FT**T**TF
T &F &T**F**F &TF**F**FT
F &F &F**T**F &TF**T**TF

Consider the following argument:

If lightning struck, we heard a loud noise.
If we heard a loud noise, lightning struck.

L = "Lightning struck."
H = "We heard a loud noise."

$L \supset H$
$H \supset L$

Is this valid? No

H &L &$L \supset H$ &$H \supset L$
T &T &T**T**T &T**T**T
F &T &T**F**F &F**T**T
T &F &F**T**T &T**F**F
F &F &F**T**F &F**T**F

13.7. Argument Forms for Conditionals. There are four simple but important argument forms that use conditionals. Here are their instances.

Modus Ponens:

There is gold in the mountain.
If there is gold in the mountain, I will be rich.
Thus, I will be rich.

which is valid.

A &C &$A \supset C$ &A &C
T &T &T**T**T &**T** &**T**
F &T &F**T**T &**F** &**T**
T &F &T**F**F &**T** &**F**
F &F &F**T**F &**F** &**F**

Denying the antecedent:

There is no gold in the mountain.
If there is gold in the mountain, I will be rich.
Thus, I will not be rich.

which is invalid.

A &*C* &*A* ⊃ *C* &¬*A* &¬*C*
T &T &T**T**T &**F**T &**F**T
F &T &F**T**T &**T**F &**F**T
T &F &T**F**F &**F**T &**T**F
F &F &F**T**F &**T**F &**T**F

Affirming the consequent:

I will be rich.
If there is gold in the mountain, I will be rich.
Thus, there is gold in the mountain.

which is invalid.

A &*C* &*A* ⊃ *C* &*A* &*C*
T &T &T**T**T &**T** &**T**
F &T &F**T**T &**T** &**F**
T &F &T**F**F &**F** &**T**
F &F &F**T**F &**F** &**F**

Modus Tollens:

I will not be rich.
If there is gold in the mountain, I will be rich.
Thus, there is no gold in the mountain.

which is valid.

A &*C* &*A* ⊃ *C* &¬*C* &¬*A*
T &T &T**T**T &**F**T &**F**T
F &T &F**T**T &**F**T &**T**F
T &F &T**F**F &**T**F &**F**T
F &F &F**T**F &**T**F &**T**F

Why is it tempting to use the invalid argument forms?

Reason I: Often there is a conversational implication that the conditional is really an 'if and only if'. Example: "If you don't brush your teeth, your teeth will rot out."

Reason II: Sometimes it's good scientific reasoning to use reasoning that looks like affirming the consequent. The following argument is the kind of argument we would say is normally reasonable in ordinary circumstances.

Joni had marijuana breath, was giggling a lot, and had "the munchies."
If Joni were stoned on marijuana, she would have marijuana breath,
would be giggling a lot, and would have the munchies.
Thus, Joni is probably stoned.

13.8. Paradoxes of the Material Conditional. Is the following statement a tautology, a contradiction, or a contingent statement?
"If a meteor will hit Australia in 2014, then Napoleon's second favorite color was yellow, or if Napoleon's second favorite color was yellow, then a meteor will Australia in 2014."

$(A \supset Y) \vee (Y \supset A)$

A &Y &$(A \supset Y) \vee (Y \supset A)$
T &T &TTT **T** TTT
F &T &FTT **T** TFF
T &F &TFF **F** FTT
F &F &FTF **T** FTF

It's a tautology.

"I am 99% confident that my sister's middle name is Angela. I conclude from this: if my sister's middle name is not Angela, then Godzilla destroyed Tokyo in 1994."

My sister's middle name is Angela.
Thus, if my sister's middle name is not Angela, then Godzilla destroyed Tokyo in 1994.

$\underline{A}$
$\neg A \supset G$

A &G &A &$\neg A \supset G$
T &T &**T** &FT **T** T
F &T &**F** &TF **T** T
T &F &**T** &FT **T** F
F &F &**F** &TF **F** F

It's valid.

It is true that Yuri Gagarin was a cosmonaut. We conclude from this: if Yuri Gagarin was born in 1382 AD, he was a cosmonaut.

$\underline{C}$
$Y \supset C$

Y &C &C &$Y \supset C$
T &T &**T** &T **T** T
F &T &**T** &F **T** T
T &F &**F** &T **F** F
F &F &**F** &F **T** F

It's valid.

13.9. Robustness. How can we explain the discrepancy between arguments that are valid when we use '⊃' to translate conditionals, but that are intuitively dumb inferences to make?

A theory proposed by Frank Jackson says that the truth table for English conditionals is the truth table for '⊃'. The difference between them is that the English conditional expresses something else in addition to truth.

There are two things we express when we utter an English conditional: truth and **robustness**.

When I say "If A, then C," I am signaling to you that
1. I believe the sentence $A \supset C$ is true.
2. I will still believe the sentence $A \supset C$ is true even if I come to find out that A is true.

If condition 2 holds, we say that $A \supset C$ is robust with respect to learning that A.

With regard to the argument,

My sister's middle name is Angela.
Thus, if my sister's middle name is not Angela, then Godzilla destroyed Tokyo in 1994.

although the conclusion follows validly using classical logic, the conclusion is not robust because even if I find out that my sister's middle name is Beth, I am going to disbelieve the premise of the argument, and then it will not lead me to accepting the conclusion that Godzilla destroyed Tokyo in 1994 if my sister's middle name is not Angela.

With regard to the argument,

Yuri Gagarin was a cosmonaut.
Thus, if Yuri Gagarin was born in 1382 AD, he was a cosmonaut.

Although the conclusion follows validly and I therefore believe it is true, it is not robust because if I find out (to my surprise) that Yuri Gagarin was born in 1382 AD, I am going to use my background knowledge that space travel didn't exist in 1382, and conclude that the premise is false. Then, the argument will not motivate me to accept the conclusion that if Yuri Gagarin was born in 1382 AD, he was a cosmonaut.

Frank Jackson's theory isn't a cheap trick cooked up to fix the paradoxical of the material conditional. It works for disjunction too.

When I say "A or B," I am signaling to you that 1. I believe the sentence $A \vee B$ is true. 2. I will still believe the sentence $A \vee B$ even if I come to find out $\neg A$. 3. I will still believe the sentence $A \vee B$ even if I come to find out $\neg B$.

With regard to the argument

I took a shower this morning.
Thus, either I took a shower this morning, or I took a casual swim in a lake of molten lava.

Although the conclusion follows validly and I therefore believe it is true, it is not robust because if I find out (to my surprise) that I didn't take a shower this morning, I am not going to believe that I was swimming in lava.

We often don't use the following argument form:

F
Thus, F *or* G.

There are two theories that try to explain this: Paul Grice says we often don't do it because of conversational conventions. If we are in a position to be more informative, we ought to be more informative. Saying 'F or G' is generally less informative than saying 'F'. Frank Jackson says we often don't do it because the conclusion is often not robust. If you believe 'F', there is no reason in general to believe 'If not F, then G'.

CHAPTER 3

Truth Trees

Truth trees (also called semantic tableaux) are a method of determining consistency for sets of statements. They are usually are more efficient than truth tables, depending on the sentences involved. another benefit of using truth trees over truth tables is that we can continue to use them when we expand our logic to include quantifiers.

In our system there are 7 rules, illustrated in Fig. **??**.

(1) **Double Negation** From any sentence of the form $\neg\neg\alpha$, derive α.
(2) **Conjunction** From any sentence of the form $(\alpha\&\beta)$, derive α and derive β.
(3) **Negation of Conjunction** From any sentence of the form $\neg(\alpha\&\beta)$, create two branches, one for $\neg\alpha$ and another for $\neg\beta$.
(4) **Disjunction** From any sentence of the form $(\alpha \vee \beta)$, create two branches, one for α and another for β.
(5) **Negation of Disjunction** From any sentence of the form $\neg(\alpha\vee\beta)$, derive $\neg\alpha$ and derive $\neg\beta$.
(6) **Conditional** From any sentence of the form $(\alpha \supset \beta)$, create two branches, one for $\neg\alpha$ and another for β.
(7) **Negation of Conditional** From any sentence of the form $\neg(\alpha \supset \beta)$, derive α and derive $\neg\beta$.

To create the truth tree, we follow these rules:

(1) Start by vertically listing all the statements whose consistency you want to check.
(2) Pick any complex statement you like that is in an open branch of the tree, and apply the rule appropriate to that statement. Strategy Tip: Do the non-branching rules first.
(3) Check for explicit inconsistencies along branches of the tree. An explicit inconsistency is when a formula and its negation appear on the same branch. If you find an explicit inconsistency, close off the branch by writing an $\times$ at the bottom of the branch where the inconsistency appears.
(4) Go to step two, unless either all branches are closed, in which case the set is inconsistent, or there is a branch where you have run out of complex statements, in which case the set is consistent.
(5) Number every line that has a statement.
(6) Mark each derived line with the number of the line from which you derived it and the rule you used.

FIGURE 1. Sentential Truth Tree Rules

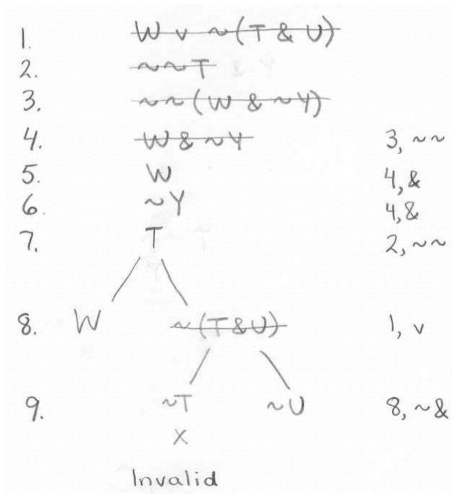

FIGURE 2. Answer to Exercise **??**

(7) Mark at the bottom whether the set is consistent or inconsistent.

If we are checking for the validity of an argument, remember that we initially list the negation of the conclusion along with the premises. A fully closed tree means the argument is valid, and a tree with an open branch means the argument is invalid. Counterexamples can be found by looking at the atomic sentences on any one open branch.

1. Truth Tree Examples

EXERCISE 12. *Use a truth tree to determine whether the argument is valid.*

$W \vee \neg(T\&U)$
$\underline{\neg\neg T}$
$\neg(W\&\neg Y)$

One counterexample to the argument in exercise **??** (corresponding to the left-most branch of the tree) is a world where W and T are true and Y is false.

EXERCISE 13. *Use a truth tree to determine whether the argument is valid.*

$\neg J \vee \neg K$
$L\&\neg M$
$\underline{(K\&L) \supset (J \vee O)}$
O

The only counterexample to the argument in exercise **??** is where L is true and O, M, and K are false.

EXERCISE 14. *Use a truth tree to determine whether the argument is valid.*

$E \supset \neg(D\&C)$
$(E \vee C) \supset D$
$\underline{B\&\neg C}$
$D \supset (B\&C)$

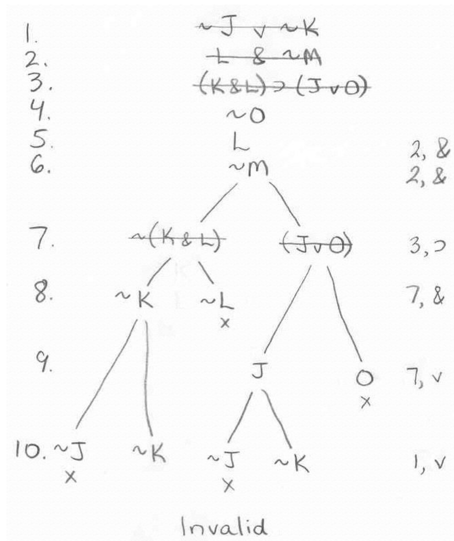

FIGURE 3. Answer to Exercise ??

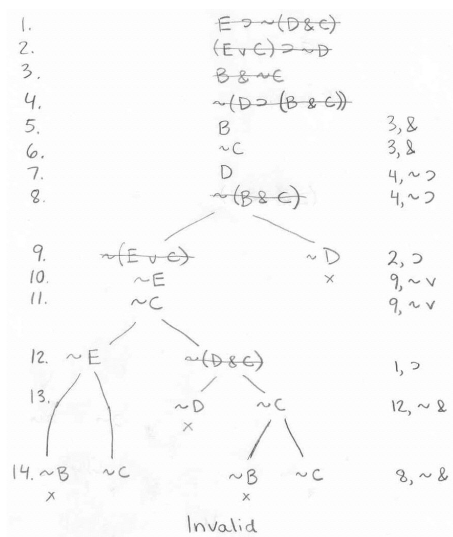

FIGURE 4. Answer to Exercise ??

One of the counterexamples to the argument in exercise **??** is where B and D are true and C and E are false.

EXERCISE 15. *Use a truth tree to determine whether the argument is valid.*

$$\begin{array}{l} (F\&G) \vee (H\&I) \\ \underline{(G\&H) \vee (F\&I)} \\ (G\&I) \vee (F\&H) \end{array}$$

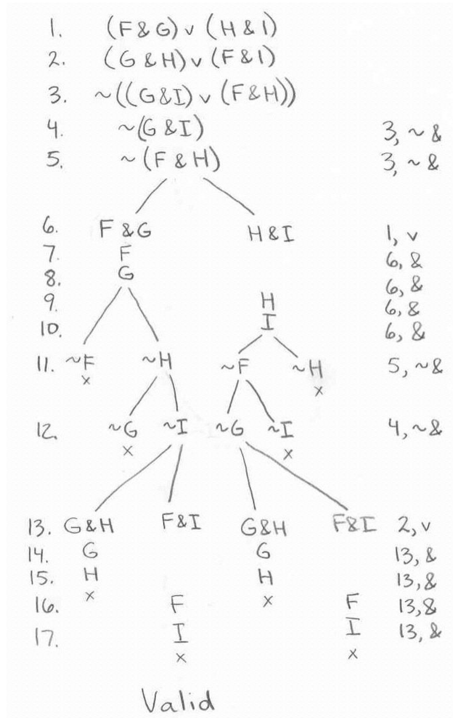

FIGURE 5. Answer to Exercise ??

CHAPTER 4

Quantifier Logic

1. The Quantifier Language

The language of quantifier logic is built on that of Sentential (Propositional) Logic.

(1) First, we add lowercase letters to our language.
(2) Contants: lowercase letters from 'a' to 't' will refer to individuals.
(3) Variables: lowercase letters from 'u' to 'z' will be variables that will only appear within the scope of a quantifier symbol.
(4) Second, we add uppercase letters that are immediately followed by lowercase letters, e.g. Px, Rab, $Bycz$, and these denote properties and relations.
(5) Third, we add quantifier symbols $\forall$ and $\exists$, which are always followed by a single variable (or in the shorthand that will be allowed, multiple distinct variables).

Other logic systems mark the universal quantifier by putting parentheses around the variable: $(x)(Mx \supset Ax)$ and the existential quantifier as $(\exists y)(Cy \& By)$.

With the new elements of our language, we need new rules for wffs.

- Any capital letter is a wff.
- **Any capital letter immediately followed by n lowercase letters (either individuals or variables) is a wff.**
- If α is a wff, then $\neg\alpha$ is a wff.
- If α is a wff, and β is a wff, then $(\alpha\&\beta)$ is a wff.
- If α is a wff, and β is a wff, then $(\alpha \vee \beta)$ is a wff.
- If α is a wff, and β is a wff, then $(\alpha \supset \beta)$ is a wff.
- **If α is a wff and x is a variable, then $\forall x\alpha$ is a wff.**
- **If α is a wff and x is a variable, then $\exists x\alpha$ is a wff.**
- Nothing else is a wff.

As shorthand, we can abbreviate $\forall x\forall y\forall z$ as $\forall xyz$, and $\exists w\exists x$ as $\exists wx$. It doesn't matter if you switch the order of the variables.

We need a rule to define the connection between wffs and sentences. We say that an instance of a variable is **bound** if it is inside the scope of a quantifier with the same kind of variable (same letter). Otherwise we say that the instance of the variable is **free**. A wff is a sentence if and only if all its variables are bound.

EXERCISE 16. *Which of the following are sentences?*

(1) $\forall x(My \supset Ax)$
(2) $\neg\exists z(Mz\&Ah)\&Mz$
(3) $\exists x\forall y(Ky \supset (\neg Ax\&\neg Q))$
(4) $\exists x\exists z(Rxz\&Ex)$
(5) $\forall x\neg\forall y\forall z(Rxy \supset \neg Hj)$

(6) $\exists x \forall z(Wx \supset Sd)$
(7) $\forall z(Rz \supset \neg\neg Fz) \supset \neg\forall x(Rz \supset Sx)$
(8) $\exists x \forall \neg z(Cxz \supset Czx)$
(9) $\forall yx \neg(My \supset Adxd)$
(10) $\exists z(Fz \& Gz) \supset \exists z(Rz \supset Sz)$
(11) $\exists x(Tx \& \forall x(Cxx \supset K))$

Answers: 3,4,5,6,9,10,11. In 1, y is free. In 2, the z at the end is not within the scope of a quantifier. In 7, the rightmost z is free. 8 is not even a wff. 6 and 11 have quantifiers that don't do anything because the quantified variable does not appear inside the scope of the quantifier, but they are legitimate sentences nonetheless.

2. Predicate Logic Truth Tree Rules

Negated Quantifier Rules:
($\neg\exists$): From $\neg\exists x(\alpha)$, derive $\forall x(\neg\alpha)$.
($\neg\forall$): From $\neg\forall x(\alpha)$, derive $\exists x(\neg\alpha)$.
Existential Quantifier Rule ($\exists$):
From $\exists x(\alpha)$, derive α with every instance of x in α replaced by a new constant.
Universal Quantifier Rule ($\forall$):
From $\forall x(\alpha)$, derive α with every instance of x in α replaced by a constant. Don't cross out the line with $\forall x(\alpha)$.
Strategies:

- Your first priority is to push any negations inside the quantifiers. Use the $\neg\exists$ and $\neg\forall$ rules to move negations from the left side of quantifiers to the right side. If you have the sentence
- Your second priority is to do the $\exists$ rule before the $\forall$ rule whenever possible.
- When you use $\forall$ on a statement, pick individuals that already appear in its branch of the truth tree, and target those choices so that the instantiated variable matches an existing constant. For example, if you already have $Rakh$ in your branch, and you are instantiating something of the form $\forall x((Ryxz \& \dots) \supset)$, you want to instantiate with k.

3. Predicate Translations

The way to translate the sentence, "Bill is acting odd." is to let

b = Bill
$Ox = x$ is acting odd.

and write the sentence as Ob.

The way to translate the sentence, "Fiona will be at home sometime during the next week," is to let

f = Fiona
$Hx = x$ will be at home sometime during the next week.

and write the sentence as Hf.

The way you do the translation is to check whether the subject of the sentence refers to an individual. If it does, translate the subject as a lowercase letter. If not, you will have to fall back on the other rules we have for translation in propositional logic. Assuming the subject of the sentence is an individual, translate what is said about that individual as a property. This will work for just about any simple sentence, even if you don't normally think of the sentence as talking about the properties of individuals. It also works for a lot of complicated sentences.

"The most repulsive man on Earth won the shock jock's prize." translates as

$r =$ the most repulsive man on Earth
$Wx = x$ won the shock jock's prize.
Wr

"The only person on the bench was sleeping." translates as

$p =$ the only person on the bench
$Sx = x$ was sleeping.
Sp

"Rosco should have done it." translates as

$r =$ Rosco
$Sx = x$ should have done it.
Sr

"Randy groomed his pet." translates as

$r =$ Randy
$Px = x$ groomed his or her pet.
Pr

Of course, it's not always that easy. If the sentence has the kind of logical structure that we saw in propositional logic, we have to capture that as well. The technique used is exactly the technique that you have already learned. First, reformulate the sentence by separating independent clauses with 'and' and 'or' and pull negations out of the sentence where possible with 'it is not the case that'. Then, convert sentences with individuals as subjects into properties and individuals.

"The last person in line won't get into the show." translates as, "It is not the case that the last person in line will get into the show." which translates as

$l =$ the last person in line.
$Sx = x$ will get into the show.
$\neg Sl$

"Dave and Martha are top prize winners in this year's competition." translates as "Dave is a top prize winner in this year's competition, and Martha is a top prize winner in this year's competition." which translates as

$d =$ Dave
$m =$ Martha
$Wx = x$ is a top prize winner in this year's competition.
$Wd \& Wm$

"The horseman approached us quickly but silently." translates as "The horseman approached us quickly, and the horseman approached us silently." which translates as

$h =$ the horseman
$Qx = x$ approached us quickly.
$Sx = x$ approached us silently.
$Qh \& Sh$

"The governor was lambasted and ridiculed for what he said." translates as "The governor was lambasted for what he said, and the governor was ridiculed for what he said." which translates as

g = the governor
$Lx = x$ was lambasted for saying what he or she said.
$Rx = x$ was ridiculed for saying what he or she said.
$Lg\&Rg$

"Janie and Gillian were pleased with the decision, but Ed wasn't." translates as "Janie was pleased with the decision, and Gillian was pleased with the decision, and it is not the case that Ed was pleased with the decision." which translates as

j = Janie
g = Gillian
e = Ed
$Px = x$ was pleased with the decision.
$(Pj\&Pg)\&\neg Pe$

"If Elena ate the rice I left in the fridge, she is going to be sick." translates as

e = Elena $Ax = x$ ate the rice I left in the fridge.
$Sx = x$ is going to be sick.
$Ae \supset Se$

"It is not the case that both Ray and his brother are unemployed," translates as "It is not the case that (it is not the case that Ray is employed and it is not the case that Ray's brother is employed)."

r = Ray// b = Ray's brother
$Ex = x$ is employed.
$\neg(\neg Er\&\neg Eb)$

"The engine and caboose are painted in solid blue and solid red respectively," translates as "The engine is painted in solid blue, and the caboose is painted in solid red."

e = the engine
c = the caboose
$Bx = x$ is painted in solid blue.
$Rx = x$ is painted in solid red.
$Be\&Rc$

Remember that you can translate a sentence into the individual w/ property form only if there is an individual. If the subject of the sentence is not an individual, you have to translate the sentence in the familiar propositional way.

"Goats eat everything, but my pet is a very picky eater." translates as

G = Goats eat everything. p = my pet $Px = x$ is a picky eater
$G\&Pp$

"I'm not a hero if heroes always do what's right." translates as

R = Heroes always do what's right.
m = me
$Hx = x$ is a hero.
$R \supset \neg Hm$

4. Quantifiers

We want our improved logic to capture the logic of some expressions that say something about the number (quantity) of things that have properties.

(1) "All dogs have fur."

(2) "Some flowers are fragrant."
(3) "At least one singer is blond."
(4) "Every instrument in the band is made of brass."
(5) "Whatever person is late will be left behind."
(6) "Anyone who objects will be silenced."
(7) "Nothing is going to happen without approval."
(8) "No one likes to have their eyes poked."
(9) "Not everyone is going to go for that idea."

The fundamental concepts behind these sentences are the universal quantifier that expresses 'every' or 'all' and the existential quantifier that expresses 'at least one'.

Universal Quantifier $\forall$ means 'every'.
Existential Quantifier $\exists$ means 'at least one'.

These two symbols will allow us to translate a number of different sentences and analyze a surprisingly large set of more sophisticated arguments.

4.1. Quantifiers We Don't Translate. These expressions make implications about number or quantity but are too complicated for our simple quantifier logic to handle.

(1) "Most sorority members use highlights."
(2) "A few limbs were damaged in the accident."
(3) "Few would offer that opinion."
(4) "Phoenicians were by and large sailors."
(5) "The overwhelming number of congressmen are rich."
(6) "The number of days in the week exceeds the number of quark flavors by one."
(7) "Birds fly."
(8) "Many people have tried and failed."

5. Universal Sentences

Universal sentences are translated with $\forall$. Some extremely simple sentences, sentences that make a claim about absolutely everything, are translated with $\forall$ followed by a property.

"Everything is ephemeral."
$Ex = x$ is ephemeral.
$\forall x Ex$

"All is lost."
$Lx = x$ is lost.
$\forall x Lx$

"Everything is black and dreary."
$Bx = x$ is black.
$Dx = x$ is dreary.
$\forall x(Bx \& Dx)$

"Everything is either real or isn't real."
$Rx = x$ is real.
$\forall x(Rx \vee \neg Rx)$

For almost all universal sentences, it's slightly more complicated. Step 1 is to translate into a conditional with the following idiom: "Every thing is such that if it is... then it is...."

- "All dogs have fur," translates as "Every thing is such that if it is a dog, then it has fur."
- "Every story has two sides," translates as "Every thing is such that if it is a story, then it has two sides.
- "Every tree has leaves or needles," translates as "Every thing is such that if it is a tree, then (it has leaves or it has needles)."
- "Everyone is thrilled," translates as "Every thing is such that if it is a person, then it is thrilled."
- "Everyone who's late will be turned away," translates as "Every thing is such that if it is a person and it is late, then it will be turned away."
- "All fallen fruit rots," translates as "Every thing is such that if it is a fruit and it is fallen, then it rots."

Step 2 is to translate into the form "(conditions on the subject ⊃ property)."

Every moose has antlers.
$Mx = x$ is a moose.
$Ax = x$ has antlers.
$\forall x(Mx \supset Ax)$

"All dogs have fur," translates as "Every thing is such that if it is a dog, then it has fur."
$Dx = x$ is a dog.
$Fx = x$ has fur.
$\forall x(Dx \supset Fx)$

"Every story has two sides" translates as "Every thing is such that if it is a story, then it has two sides."
$Sx = x$ is a story.
$Tx = x$ has two sides.
$\forall x(Sx \supset Tx)$

"Every tree has leaves or needles," translates as "Every thing is such that if it is a tree, then (it has leaves or it has needles)."
$Tx = x$ is a tree.
$Lx = x$ has leaves.
$Nx = x$ has needles.
$\forall x(Tx \supset (Lx \vee Nx))$

"Everyone is thrilled," translates as "Every thing is such that if it is a person, then it is thrilled."
$Px = x$ is a person.
$Tx = x$ is thrilled.
$\forall x(Px \supset Tx)$

"Everyone who's late will be turned away," translates as "Every thing is such that if it is a person and it is late, then it will be turned away."
$Px = x$ is a person.
$Lx = x$ is late.

$Tx = x$ will be turned away.
$\forall x((Px \& Lx) \supset Tx)$

"All fruit on the ground rots," translates as "Every thing is such that if it is a fruit and it is fallen, then it rots."
$Fx = x$ is a fruit.
$Gx = x$ is on the ground.
$Rx = x$ rots.
$\forall x((Fx \& Gx) \supset Rx)$

"All mud huts without permits will be condemned and demolished," translates as "Every thing is such that if it is a hut, and it is made of mud, and it is not the case that it has a permit, then it will be condemned and it will be demolished."
$Hx = x$ is a hut.
$Mx = x$ is made of mud.
$Px = x$ has a permit.
$Cx = x$ will be condemned.
$Dx = x$ will be demolished.
$\forall x(((Hx \& Mx) \& \neg Px) \supset (Cx \& Dx))$

"Every indicted suspect in this case will have his or her fingerprints on file," translates as "Every thing is such that if it is indicted and it is a suspect in this case, then it will have its fingerprints on file."
$Ix = x$ is indicted.
$Sx = x$ is a suspect in this case.
$Fx = x$ will have its fingerprints on file.
$\forall x((Ix \& Sx) \supset Fx)$

"If everyone agrees, Jack will get started," translates as "If (every thing is such that if it is a person, then it agrees), then Jack will get started."
$j =$ Jack
$Px = x$ is a person.
$Ax = x$ agrees.
$Sx = x$ will get started.
$\forall y(Py \supset Ay) \supset Sj$

6. Existential Sentences

Existential sentences are translated with $\exists$. Some simple sentences, sentences that make a claim asserting the existence of something, are translated with $\exists$ followed by a property.

"There exists a demon."
$Dx = x$ is a demon.
$\exists xDx$

"Something is lost."
$Lx = x$ is lost.
$\exists xLx$

"There is at least one universe."
$Ux = x$ is a universe.
$\exists xUx$

More common sentences make more restricted claims of existence. For these, one forms a large conjunction of all the properties of the subject whether they come from adjectives modifying the subject or are predicates following the verb.

(1) Translate into a conjunction with the following idiom: "There exists a thing such that (it is..., and it is..., and it is..., etc)."
(2) Translate the conjunction into symbols: $\exists x((Ax\&Bx)\&(Cx\&(Dx\&Ex)))$

"Some castle wall was breached," translates as "There exists a thing such that (it is a castle wall and it is breached)."
$Cx = x$ is a castle wall.
$Bx = x$ was breached.
$\exists y(Cy\&By)$

"There's a ferret in my bed," translates as "There exists a thing such that (it is a ferret and it is in my bed)."
$Fx = x$ is a ferret.
$Bx = x$ is in my bed.
$\exists x(Fx\&Bx)$

"Some child stole the candy on the back shelf of the store," translates as "There exists a thing such that (it is a child and it stole the candy on the back shelf of the store)."
$Cx = x$ is a child.
$Sx = x$ stole the candy on the back shelf of the store.
$\exists x(Cx\&Sx)$

"A red car was sold yesterday," translates as "There exists a thing such that (it is a car, and it is red, and it was sold yesterday)."
$Cx = x$ is a car.
$Rx = x$ is red.
$Sx = x$ was sold yesterday.
$\exists z((Cz\&Rz)\&Sz)$

"Someone underage will be at the strip club and will likely be getting blasted," translates as "There exists a thing such that (it is a person, and it is underage, and it will be at the strip club, and it will likely be getting blasted)."
$Px = x$ is a person.
$Ux = x$ is underage.
$Sx = x$ will be at the strip club.
$Bx = x$ will likely be getting blasted.
$\exists y((Py\&Uy)\&(Sy\&By))$

"A rather long python was seen at the scene of the crime," translates as "There exists a thing such that (it is a python, and it is rather long, and it was seen at the scene of the crime)."
$Px = x$ is a python.
$Lx = x$ is rather long.
$Sx = x$ was seen at the scene of the crime.
$\exists x((Px\&Lx)\&Sx)$

"Someone is to blame for this mess, and it's not me," translates as "There exists a thing such that (it is a person, and it is to blame for this mess), and it is not the case that I am to blame for this

mess.
$Px = x$ is a person.
$Bx = x$ is to blame for this mess.
$m =$ me
$\exists x((Px \& Bx) \& \neg Bm$
"Unicorns exist," translates as "There exists a thing such that it is a unicorn."
$Ux = x$ is a unicorn.
$\exists x Ux$

We don't worry about the fact that 'unicorns' is plural because the sentence can plausibly be interpreted as true if a single unicorn is discovered.

6.1. Any, Anyone, Anybody, Anything. Most of the time, 'any' is translated as a universal but sometimes as an existential. The way to tell is just to try to substitute 'everyone' and try to substitute 'something' and go with what sounds like a better translation. Tip: The existential translation usually comes in the antecedent of some conditional.

Suppose we are translating, "Anyone can ride a bike." We ask ourselves which sounds more accurate: "Everyone can ride a bike," or "Someone can ride a bike"? "Everyone can ride a bike," sounds better so we go ahead and translate that to "Every thing is such that (if it is a person then it can ride a bike)." and then translate that directly into symbols as $\forall x(Px \supset Bx)$.

With "If anyone comes, I will know," we ask ourselves which sounds more accurate: "If everyone comes, I will know," or "If someone comes, I will know"? Because "If someone comes, I will know," sounds better, we take it and translate it into, "If (there exists a thing such that it is a person and that thing comes), then I will know," and then into symbols as $\exists x(Px \& Cx) \supset K$.

With "Anything that exposes us to risk will be handled with aggression," we we ask ourselves which sounds more accurate: "Everything that exposes us to risk will be handled with aggression," or "Somthing that exposes us to risk will be handled with aggression." We use the 'everything' translation and spell it out as "Every thing is such that (if it exposes us to risk, then that thing will be handled with aggression)," and then as $\forall x(Rx \supset Ax)$.

With "Not just anyone can fly an airplane," we we ask ourselves which sounds more accurate: "Not everyone can fly an airplane," or "Not someone can fly an airplane." (We eliminate the 'just' because it does not seem to contribute to the meaning of the sentence, and it makes the 'everyone' version and 'someone' version awkward.) The 'someone' version says no one can fly an airplane, so we use the first version and expand it into, "It is not the case that (everyone can fly an airplane)," and then into $\neg\forall x(Px \supset Fx)$.

6.2. No, None, No one, Nobody, Nothing. These expressions translate as follows:

- 'no' = 'not some'
- 'none' = 'not one'
- 'no one' = 'not someone'
- 'nobody' = 'not somebody'
- 'nothing' = 'not something'

This means they are negations of existential claims.

"No dogs are allowed," translates as "Not some dogs are allowed," which translates as "It is not the case that (there exists a thing such that (it is a dog and it is allowed))."
$Dx = x$ is a dog.
$Ax = x$ is allowed.
$\neg\exists x(Dx\&Ax)$

"No white buffaloes can be seen today," translates as "Not some white buffaloes can be seen today," which translates as "It is not the case that (there exists a thing such that (it is a buffalo and it is white and it can be seen today))."
$Bx = x$ is a buffalo.
$Wx = x$ is white.
$Sx = x$ can be seen today.
$\neg\exists x((Bx\&Wx)\&Sx)$

"No one here is yelling and screaming," translates as "Not someone here is yelling and is screaming," which translates as "It is not the case that (there exists a thing such that (it is a person and it is here and it is yelling and it is screaming))."
$Px = x$ is a person.
$Hx = x$ is here.
$Yx = x$ is yelling.
$Sx = x$ is screaming.
$\neg\exists x((Px\&Hx)\&(Yx\&Sx))$

"There is nothing that is noble that is worth fighting for," translates as "There is not something noble that is worth fighting for," which translates as "It is not the case that (there exists a thing such that (it is noble and it is worth fighting for))."
$Nx = x$ is noble.
$Wx = x$ is worth fighting for.
$\neg\exists x(Nx\&Wx)$

"Nobody who had put up with your outbursts or had dealt with you for any length of time is going to want to celebrate your engagement," translates as "Not somebody who (had put up with your outbursts or had dealt with you for any length of time) is going to want to celebrate your engagement," which translates as "It is not the case that (there exists a thing such that (it is a person and (it had put up with your outbursts or it had dealt with you for any length of time) and it is going to want to celebrate your engagement))."
$Px = x$ is a person.
$Ox = x$ had put up with your outbursts.
$Dx = x$ had dealt with you for any length of time.
$Cx = x$ is going to want to celebrate your engagement.
$\neg\exists x((Px\&(Ox \vee Dx))\&Cx)$

6.3. Alternative Formulation of No, None, Nobody, etc. Instead of translating these expressions as existentials, you can express them equivalently as universals.

"None of Carrot-top's jokes are funny," translates as "Everything is such that (if it's a Carrot-top joke, then it's not funny)."
$Jx = x$ is a Carrot-top joke.
$Fx = x$ is funny.
$\forall x(Jx \supset \neg Fx)$
It is logically equivalent to $\neg\exists x(Jx\&Fx)$.

"No one likes you and nobody wants to be seen around you," translates as "Everything is such that (if it is a person, then it does not like you), and everything is such that (if it is a person, then it does not want to be seen around you)."
$Px = x$ is a person.
$Lx = x$ likes you.
$Wx = x$ wants to be seen around you.
$\forall x(Px \supset \neg Lx)\&\forall(Px \supset \neg Wx)$
It is logically equivalent to $\forall x(Px \supset (\neg Lx\&\neg Wx))$ and $\neg\exists x(Px\&(Lx\&Wx))$.

6.4. Not All vs. Not Any. Be careful about where you put the negation.
"Not all of the people in this room are rude."
$\neg\forall x(Px \supset Rx)$
"Not any of the people in this room are rude," translates as "Not some of the people in this room are rude."
$\neg\exists x(Px\&Rx)$

6.5. Articles.

"The squirrel is in the attic."
As

"The squirrels are playing."
This is ambiguous between 'most of the squirrels are playing', and 'all the squirrels are playing'. It also seems to imply plurality, which we can't do. The first possibility we can't do yet. The second possibility is $\forall x(Sx \supset Px)$

"A squirrel is in the attic."
This is ambiguous between 'there is exactly one squirrel in the attic', and 'there is at least one squirrel in the attic'. The first possibility we can't do yet. The second possibility is $\exists x(Sx\&Ax)$

"A squirrel is a distant relative of the groundhog," translates as "Squirrels are distant relatives of the groundhog." $\forall x(Sx \supset Rx)$

"Squirrels are in the attic," translates as "There are squirrels in the attic." This is ambiguous between "There is more than one squirrel in the attic," and "There is at least one squirrel in the attic." $\exists x(Sx\&Ax)$

"Squirrels are mammals," translates as "All squirrels are mammals."
$\forall x(Sx \supset Mx)$

"Squirrels build nests in trees," means, "By and large, squirrels build nests in trees." We can't translate this with predicates.

6.6. Only. Remember how 'A if B' is equivalent to 'B only if A'? 'Only' sentences are universals where the direction of the horseshoe is flipped (the antecedent and consequent are exchanged).

> "Only members are allowed in the clubhouse," translates as "Every thing is such that (if it is allowed in the clubhouse, then it is a member)."
> $Cx = x$ is allowed in the clubhouse.
> $Mx = x$ is a member.
> $\forall x(Cx \supset Mx)$

Notice how with the 'only' the subject is to the right of the horseshoe, and the verb phrase is to the left.

> "Only the courageous can play," translates as "Every thing is such that (if it can play, then it is courageous)."
> $Cx = x$ is courageous.
> $Px = x$ can play.
> $\forall x(Px \supset Cx)$
>
> How do we translate, "If only members of our club were invited, then all our members came"?
> $Mx = x$ is a member of our club.
> $Ix = x$ was invited.
> $Cx = x$ came.
> $\forall x(Ix \supset Mx) \supset \forall x(Mx \supset Cx)$

"The only children who are allowed in this playground are those from Rosewood Elementary." translates as "Only children from Rosewood Elementary are allowed in this playground."

> $Cx = x$ is a child.
> $Ax = x$ is allowed to be in this playground.
> $Rx = x$ is from Rosewood Elementary.
> $\forall x((Cx \& Ax) \supset Rx)$

Note that the translation is not $\forall x(Ax \supset (Cx \& Rx))$ which is different in that it says adults are not allowed on the playground (and dogs are not allowed, and bananas are not allowed, and purple bow-ties are not allowed, etc.) Effectively, it says the only things allowed in the playground are Rosewood children. Notice how (compared to the universal claim "All children from Rosewood Elementary are allowed in this playground," which is $\forall x((Cx \& Rx) \supset Ax))$ we don't strictly have a switch of the consequent with the antecedent but only with one conjunct in the antecedent. To see what's going on, it is useful to use the so-called importation and exportation rules. Start with "All children from Rosewood Elementary are allowed in this playground," which is $\forall x((Cx \& Rx) \supset Ax))$. This is equivalent to $\forall x(Cx \supset (Rx \supset Ax))$ which says that if you are a child, then if you are from Rosewood, you are allowed. Adding the 'only' to the sentence (to give us "Only children from Rosewood Elementary are allowed in this playground,") makes us switch the Ax and Rx, without touching the Cx. This gives us $\forall x(Cx \supset (Ax \supset Rx))$ which says that if you are a child, then if you are allowed, then you are from Rosewood. This is equivalent to the correct answer above, $\forall x((Cx \& Ax) \supset Rx)$, which says that all allowed children are from Rosewood. The reason we know we should not switch the Cx along with the Rx is that in the original sentence, it says "The only children who are allowed..." which means, "Among children, the only who are allowed...." It does not mean that the only things allowed in the playground are Rosewood children. Toys are allowed, slides, parents, etc.

7. Summary of Rules for Predicate Translation

$\forall$ is for ALL, every, each. '$\forall$' uses '$\supset$' to connect the subject restrictions with the verb phrase. For example, "Every gray, furry, mangy cat is old or weak or sick," is $\forall x((Gx\&(Fx\&(Mx\&Cx))) \supset (Ox \vee (Wx \vee Sx)))$.

$\exists$ is for EXISTS, some, a, an. '$\exists$' uses '&' to connect the subject restrictions with the verb phrase. For example, "Some gray, furry, mangy cat is old or weak or sick," is $\exists x((Gx\&(Fx\&(Mx\&Cx)))\&(Ox \vee (Wx \vee Sx)))$.

- "Some S's are A" is translated with $\exists x(Sx\&Ax)$.
- "No S's are A" is translated as $\neg\exists x(Sx\&Ax)$.
- "All S's are A" is translated as $\forall x(Sx \supset Ax)$.
- "Every S is an A" is translated as $\forall x(Sx \supset Ax)$.
- "Not all S's are A" is translated as $\neg\forall x(Sx \supset Ax)$.
- "Not every S is an A" is translated as $\neg\forall x(Sx \supset Ax)$.
- Anytime you are using a word like 'someone' or 'somebody' or 'anyone' or 'antybody' or 'everyone' or 'everybody', there is an implicit restriction to people. Thus you must use a predicate that
- 'Any' could mean 'some' or 'every'. Plug in both and see what sounds better. Then use the rules for the sentence that sounds better. The times when it translates as 'some' are when it is in the antecedent of a conditional, and a few rare occasions when it is negated.
- If there is an ambiguity in the sentence, and only one disambiguation can be translated into predicate logic, pick the disambiguation that can be translated into predicate logic. E.g., "Unicorns exist," should be translated as "There exists at least one unicorn."
- Indefinite articles are sometimes $\exists$, and other times $\forall$. To know which think about whether the sentence applies to one specific object or to objects generally.
- 'Only' is translated with $\forall$, but with the usual order flipped.
- Sentences that have individuals as their subjects don't have quantifiers around the subject. Just put the predicate with the lowercase letter.
- If you have a conditional that draws a general conclusion about something, the use of 'something' is translated with a $\forall$. "If something is red then it is square," translates as $\forall x(Rx \supset Sx)$.
- If you have a conditional where neither antecedent nor consequent refer to anything in the other, translate with the '$\supset$' as the main connective. "If something is red then something is square," translates as $\exists x(Rx) \supset \exists x(Sx)$.

CHAPTER 5

Relations

Relations are the name for connections between individuals. They are different from properties only in that properties apply to just a single individual. Relations apply to more than one.

Properties apply to a single individual:

- "The swing set is broken."

Relations apply to two or more individuals:

- "Randy is in front of the bar." (2 individuals)
- "The coffee table is between the sofa and the TV." (3 individuals)
- "The goat ate the flower in my garden at o'clock." (4 individuals)

1. Translating Relations

We represent relations with a capital letter and a superscript representing how many things they connect. We use variables x, y, and z to tell us which individuals have which role in the English sentence.

$Fxy = x$ is a friend to y.
$k =$ Katy
$m =$ Michelle
$c =$ Cliff

"Katy is a friend to Michelle."

Fkm

"Cliff is a friend to Katy."

Fck

"Cliff and Michelle are friends."

$Fcm \& Fmc$.

$Bxyz = x$ is between y and z.
$j =$ Jill
$h =$ the house
$c =$ the car
$b =$ Jill's brother

"Jill is between her brother and the car."

$Bjbc$

"Jill's brother is between the house and the car."

$Bbhc$

"The car is between Jill and her brother."

$Bcjb$

"The car is between Jill's brother and Jill."

$Bcbj$

(You should not use additional facts that you know about the 'between' relation to reorder the individuals. As far as logic goes, the last two logic sentences have different meanings even though the English versions are equivalent.)

1.1. Using Quantifiers with Relations. We can quantify over relations, but we have to use a variable as a placeholder so that we know which individual in the relation is quantified.

$Oxy = x$ is older than y
$m =$ Michelle
$c =$ Michelle's car

"Michelle's car is older than she is."

Ocm

"Michelle is older than something."

$\exists x(Omx)$

"Something is older than Michelle."

$\exists x(Oxm)$

"Everything is older than Michelle's car."

$\forall x(Oxc)$

"Michelle is older than everything."

$\forall x(Omx)$

"Nothing is older than Michelle."

$\neg\exists x(Oxm)$

"Something is older than everything."

$\exists x\forall y(Oxy)$

"Everything is older than something."

$\forall x\exists y(Oxy)$

$Px = x$ is a person.
$Lxy = x$ likes y.
$b =$ Beth

"Everyone likes Beth."

$\forall y(Py \supset Lyb)$

"Not everyone likes Beth."

$\neg\forall y(Py \supset Lyb)$

"Something likes Beth."

$\exists zLzb$

"Someone likes Beth."

$\exists z(Pz \& Lzb)$

"Beth likes something."

$\exists xLbx$

"Beth likes someone."

$\exists x(Px\&Lbx)$

$Px = x$ is a person.
$Lxy = x$ likes y.
$s =$ Steve
$w =$ Wendy

"If Steve likes someone then Wendy likes someone."

$\exists z(Pz\&Lsz) \supset \exists y(Py\&Lwy)$

"Unless Wendy likes Steve, Wendy doesn't like anyone."

$\neg Lws \supset \neg\exists y(Py\&Lwy)$

"Either Steve likes someone or Wendy likes everyone."

$\exists z(Pz\&Lsz) \vee \forall y(Py \supset Lwy)$

"Everyone that Steve likes, Wendy likes too."

$\forall y((Py\&Lsy) \supset Lwy)$

"Anyone that likes Steve, is liked by Wendy and Steve both."

$\forall y((Py\&Lys) \supset (Lwy\&Lsy))$

"If no one likes Wendy, then Wendy doesn't like everyone."

$\neg\exists z(Pz\&Lzw) \supset \neg\forall y(Py \supset Lwy)$

"Wendy likes everyone who likes her in return."

$\forall y((Py\&Lyw) \supset Lwy)$

"Wendy and Steve like the same people."

$\forall y((Py\&Lwy) \supset Lsy)\&\forall y((Py\&Lsy) \supset Lwy)$
$\forall y(Py \supset ((Lwy \supset Lsy)\&(Lsy \supset Lwy)))$

"Everyone likes someone."

$\forall x(Px \supset \exists y(Py\&Lxy))$

2. Translating Multiply Nested Quantified Relations

$Kxy = x$ knows y.
$Px = x$ is a person.
$Ex = x$ has an education.

"Everyone who has an education is known to someone who knows Paul."

Stage I: Write down the predicates and relations, whether simple or complex, marking the related individuals with the appropriate quantifiers. Put the subscripts in the order that makes sense given the meaning of the relation. Use different variables for each subscript.

$K_{\exists x(Px\&Kxp),\forall y(Py\&Ey)}$

Stage II: Pull the quantifiers from the subscript position out in front of the relation, using the appropriate connective, $\supset$ for $\forall$ and & for $\exists$. Do this in the order given by the English sentence. (The rule about English order does not always work, but is a reasonably good rule of thumb.)

$\forall y((Py\&Ey) \supset K_{\exists x(Px\&Kxp),y})$

which then becomes

$\forall y((Py\&Ey) \supset \exists x((Px\&Kxp)\&Kxy))$

and because we have no more subscripts for K we are done.

$Rxy = x$ is richer than y.
$Px = x$ is a person.
$Ox = x$ owns a car.

"Anyone who owns a car is richer than anyone who doesn't."
"Everyone who owns a car is richer than everyone who doesn't."

$R_{\forall x(Px\&Ox),\forall y(Py\&\neg Oy)}$
becomes
$\forall x((Px\&Ox) \supset R_{x,\forall y(Py\&\neg Oy)})$
becomes
$\forall x((Px\&Ox) \supset \forall y((Py\&\neg Oy) \supset Rxy))$

$Bxyz = x$ sits between y and z.
$Px = x$ is a person.
$Sx = x$ snores.
$Cx = x$ coughs.

"No one who snores sits between someone who coughs and someone who doesn't cough."
"There does not exist someone who snores and sits between someone who coughs and someone who doesn't cough."

$B_{\neg\exists x(Px\&Sx),\exists y(Py\&Cy),\exists z(Pz\&\neg Cz)}$
becomes
$\neg\exists x((Px\&Sx)\&B_{x,\exists y(Py\&Cy),\exists z(Pz\&\neg Cz)})$
$\neg\exists x((Px\&Sx)\&\exists y((Py\&Cy)\&B_{x,y,\exists z(Pz\&\neg Cz)}))$
$\neg\exists x((Px\&Sx)\&\exists y((Py\&Cy)\&\exists z((Pz\&\neg Cz)\&Bxyz)))$

$Cxy = x$ cares for y.
$Vx = x$ is a veterinarian.
$Px = x$ is a professional.
$Ax = x$ is an amateur.
$Fx = x$ has fleas.
$Tx = x$ has ticks.
$Dx = x$ is a dog.

"Any dog who has fleas or ticks is cared for by some veterinarian whether professional or amateur."
"Every dog who has fleas or ticks is such that there is some veterinarian who is either professional or amateur who cares for that dog."

$C_{\exists x(Vx\&(Px\vee Ax)),\forall y(Dy\&(Fy\vee Ty))}$
$\forall y((Dy\&(Fy \vee Ty)) \supset C_{\exists(Vx\&(Px\vee Ax)),y})$
$\forall y((Dy\&(Fy \vee Ty)) \supset \exists x((Vy\&(Fx \vee Tx))\&Cxy))$

$Gxy = x$ gives to y.
$Bxy = x$ is better than y.
$Px = x$ is a person.

"Anyone who gives to someone is better than everyone who doesn't give to anyone."
"Every person who gives to some person is better than every person who doesn't give to some person."

$B_{\forall x(Px\&Gx\exists zPz),\forall y(Py\&\neg Gy\exists zPz)}$

$\forall x((Px\&G_{x\exists zPz}) \supset B_{x,\forall y(Py\&\neg G_{y\exists zPz})})$
$\forall x((Px\&G_{x\exists zPz}) \supset \forall y((Py\&\neg G_{y\exists zPz}) \supset Bxy)))$
$\forall x((Px\&\exists z(Pz\&Gxz)) \supset \forall y((Py\&\neg\exists z(Pz\&Gyz)) \supset Bxy)))$

CHAPTER 6

Identity

The resources we have in our logic do not allow us to express identity, that an object under one name and an object under another name can be said to be the same object. To see why our logic is incapable of expressing identity, let's explore three different strategies for expressing identity using properties, and we will see how each strategy has inadequacies. Let's try to translate, "The chancellor is Elaine Coleman."

In the first strategy, we symbolize a property of being Elaine Coleman.

$c =$ the chancellor
$Ex = x$ is Elaine Coleman.

The sentence translates as Ec. One problem with this way of symbolizing identity is that

<u>*The chancellor is Elaine Coleman.*</u>
Thus, Elaine Coleman is not the chancellor's administrative assistant.

looks valid, but

$\underline{Ec}$
$\neg Ea$

is invalid.

In the second strategy, we symbolize a property of being the chancellor.

$Cx = x$ is the chancellor.
$e =$ Elaine Coleman

The sentence translates as Ce. One problem with this way of symbolizing identity is that

<u>*The chancellor is Elaine Coleman.*</u>
Thus, Harvey Kemp is not the chancellor.

looks valid, but

$\underline{Ce}$
$\neg Ch$

is invalid.

In the third strategy, we symbolize both properties.

$Cx = x$ is the chancellor.
$Ex = x$ is Elaine Coleman.

The sentence translates as $\forall x((Cx \supset Ex) \& (Ex \supset Cx)) \& \exists x Ex$

One problem with this way of symbolizing identity is that

<u>*The chancellor is Elaine Coleman.*</u>
Thus, no one other than Elaine Coleman is the chancellor.

looks valid, but we have no way to translate the conclusion.

What is missing in our translation is the ability to capture the difference between identity and predication. Consider different uses of the verb 'to be':

- Elaine is the chancellor.
- Elaine is tired.
- Elaine is my mother.
- Elaine is overwhelmed by my project.
- Elaine is blonde.
- Elaine is a bureaucrat.

In some cases, we are using 'is' to say that Elaine is identical with some individual. In other cases, we are using 'is' to say that Elaine has some property or quality or relation. In order to strengthen our language so that it can make the distinction, we introduce a new symbol '=', and some new rules.

First, we introduce one new rule into our definition of wff's.

- Any capital letter is a wff.
- Any capital letter immediately followed by n lowercase letters (either individuals or variables) is a wff.
- If α is a wff, then $\neg\alpha$ is a wff.
- If α is a wff, and β is a wff, then $(\alpha\&\beta)$ is a wff.
- If α is a wff, and β is a wff, then $(\alpha \vee \beta)$ is a wff.
- If α is a wff, and β is a wff, then $(\alpha \supset \beta)$ is a wff.
- If α is a wff and x is a variable, then $\forall x\alpha$ is a wff.
- If α is a wff and x is a variable, then $\exists x\alpha$ is a wff.
- $(a = b)$ **is a wff if** a **and** b **are variables or constants.**
- Nothing else is a wff.

We don't need to modify our definition of a statement as a wff with no free variables. In addition, we can abbreviate anything of the form $\neg(a = b)$ as $a \neq b$. Also, to cut down on the number of parentheses, you can drop the parentheses around an equality.

1. Truth Trees with Identity

To address our new symbol '=' we need two new rules.

- Substitution of Identicals (=): From any sentence of the form $a = b$ and another sentence that includes a, derive a sentence that is made by replacing as many instances of a as you like with b. (Or replace as many b's as you like with a). Cross out the sentence that you replaced.
- Any instance of $(a \neq a)$ counts as an explicit contradiction, where a is any term.

EXAMPLE 5. *Substitution of Identicals*

1.	$\exists x(Kx\&(Sx \vee (Mc\&Rxc)))$	Premise
2.	$c = f$	Premise
3.	$\exists x(Kx\&(Sx \vee (Mc\&Rxf)))$	1, 2, =

EXAMPLE 6. *Identity*

1.	$\exists x(x \neq g \& Fx)$	Premise
2.	$\forall xy(x = y)$	Premise
3.	$b \neq g \& Fb$	1,∃
4.	$b \neq g$	3,&
5.	Fb	3,&
6.	$b = g$	2,∀
7.	$g \neq g$	4,6,=
	$\times$	

2. Translations with Identity

2.1. Exceptions.

Ux = x is upset.
Px = x is a person.
d = Dirk

"Someone other than Dirk is upset."
There exists a person who isn't Dirk and who is upset.
$\exists x((Px \& x \neq d) \& Ux)$

"Everyone is upset except possibly Dirk."
Every person who isn't Dirk is upset.
$\forall x((Px \& x \neq d) \supset Ux)$

"Everyone except Dirk is upset."
Every person who isn't Dirk is upset.
$\forall x((Px \& x \neq d) \supset Ux)$

"Everyone besides Dirk is upset."
Every person who isn't Dirk is upset.
$\forall x((Px \& x \neq d) \supset Ux)$

"Everyone is upset except Dirk."
Every person who isn't Dirk is upset, and Dirk is upset.
$\forall x((Px \& x \neq d) \supset Ux) \& \neg Ud$

"Dirk is the only person who is upset."
Dirk is upset, and every person who isn't Dirk is not upset.
$Ud \& \forall x((Px \& x \neq d) \supset \neg Ux)$

"Only Dirk is upset."
Dirk is upset, and every person who isn't Dirk is not upset.
$Ud \& \forall x((Px \& x \neq d) \supset \neg Ux)$

2.2. Superlatives.

Txy = x is taller than y.
Mx = x is a mountain.
e = Mount Everest
k = K2

"Mount Everest is the tallest mountain."
Mount Everest is a mountain, and every mountain except Mount Everest is such that Mount Everest is taller than it.
$Me\&\forall x((Mx\&x \neq e) \supset Tex)$

"Mount Everest is the next tallest mountain after K2."
Mount Everest is a mountain, and K2 is a mountain, and K2 is taller than Mount Everest, and every mountain that isn't K2 and isn't Mount Everest is such that Mount Everest is taller than it.
$(Me\&Mk)\&\forall x((Mx\&x \neq k) \supset Tkx)\&\forall x((Mx\&x \neq e\&x \neq k) \supset Tex)$

2.3. Counting. The identity relation allows us to symbolize quantitative features. Specifically, it allows us to express the concepts of 'more than', 'fewer than', 'an equal number of', and to include specific numbers in these relations to express claims like, "There are exactly five birds flying," and "There are no more than three people in the room."

To express "There are at least n objects with the properties P, Q, and R," the pattern is to set up n variables under an existential quantifier and then inside the scope of the quantifier to state (1) that each of the n variables has the properties P, Q, and R, and (2) that none of the variables is equal to any other. One must conjoin every pair of quantified variables in an inequality.

$Ax = x$ was arrested.
$Px = x$ is a person.

"At least one person was arrested."
There exists an x who is a person and arrested.
$\exists x(Px\&Ax)$

"At least two people were arrested."
There exists an x and a y where x is a person and arrested; and y is a person and arrested and y not the same as x.
$\exists xy(Px\&Ax\&Py\&Ay\&y \neq x)$

"At least three people were arrested."
There exists an x and a y and a z where x is a person and arrested; and y is a person and arrested and is not the same as x; and there exists a z who is a person and is arrested and is not the same as x and is not the same as y;
$\exists xyz(Px\&Ax\&Py\&Ay\&Pz\&Az\&y \neq x\&z \neq x\&z \neq y)$

"At least four people were arrested."
There exists an x who is a person and is arrested; and there exists a y who is a person and is arrested and is not the same as x; and there exists a z who is a person and is arrested and is not the same as x and is not the same as y; and there exists a w who is a person and is arrested and is not the same as x and is not the same as y and is not the same as z;

$\exists xyzw(Px\&Ax\&Py\&Ay\&Pz\&Az\&Pw\&Aw\&x \neq y\&z \neq x\&z \neq y\&w \neq x\&w \neq y\&w \neq z)$

To express “There are at most n objects with the properties P, Q, and R,” the pattern is to set up $n+1$ variables under a universal quantifier and then inside the scope of the quantifier to state a conditional with (1) the antecedent that each of the $n+1$ variables has the properties P, Q, and R, and (2) the consequent that (at least) two of the variables are equal to each other. One must disjoin every pair of quantified variables in an inequality in the consequent.

“At most one person was arrested.”
For every person x and every person y, if x is a person and arrested and y is a person and arrested, then they are the same person.
$\forall xy((Px\&Ax\&Py\&Ay) \supset x = y)$

“At most two people were arrested.”
For every person x and every person y and every person z, if x is a person and arrested and y is a person and arrested and z is a person and arrested, then either x is the same as y or y is the same as z or x is the same as z.
$\forall xyz((Px\&Ax\&Py\&Ay\&Pz\&Az) \supset (x = y \lor x = z \lor y = z))$

“At most three people were arrested.”
For every person x and every person y and every person z and every person w, if x is a person and arrested and y is a person and arrested and z is a person and arrested and w is a person and arrested, then either x is the same as y or y is the same as z or x is the same as z or x is the same as w or y is the same as w or z is the same as w.
$\forall xyzw((Px\&Ax\&Py\&Ay\&Pz\&Az\&Pw\&Aw) \supset (y = x \lor z = x \lor z = y \lor w = x \lor w = y \lor w = z))$

To express “There are exactly n objects with the properties P, Q, and R,” one just conjoins the expressions for “there are at least n...,” and “there are at most n....”

“Exactly one person was arrested.”
At least one person was arrested and at most one person was arrested.
$\exists x(Px\&Ax)\&\forall xy((Px\&Ax\&Py\&Ay) \supset x = y)$

“Exactly two people were arrested.”
At least two people were arrested and at most two people were arrested.
$\exists xy(Px\&Ax\&Py\&Ay\&y \neq x)\&\forall xyz((Px\&Ax\&Py\&Ay\&Pz\&Az) \supset (x = y \lor x = z \lor y = z))$

Although we have the resources to make quantitative statements, we don't yet have the ability to do arithmetic. Arithmetic requires our adding functions to our logic.

2.4. Infinity. Now that we can express the concept of equality in our logic, we can create a set that is true in all infinite models and false in all finite models. This is just the set that contains all the expressions of the form "There are at least n things." The set would look like

$\{\exists xy(y \neq x),$
$\exists xyz(y \neq x \& z \neq x \& z \neq y),$
$\exists xyzw(x \neq y \& z \neq x \& z \neq y \& w \neq x \& w \neq y \& w \neq z),$
etc.$\}$

While we do have a set that is true iff the model is infinite, we cannot yet express with a single sentence in our logic the idea that there are an infinity of things, nor can we express the idea that there are a finite number of things. This requires what is called second order logic.

3. Implicit Properties of Binary Relations

Some concepts have built in qualities that our quantifier logic with identity can uncover. For example, the argument

K2 is taller than Mount Everest.
Thus, Mount Everest is not taller than K2.

is valid. But when we translate it as

$\underline{Tke}$
$\neg Tek$

we get an invalid argument. The reason is that the validity of the natural language version hinges on the meaning of 'tall' which our straightforward translation does not capture. Luckily in this case, the relevant aspect of the tallness relation can be expressed in our logic. We need to express the idea that tallness is anti-symmetric, that if x is taller than y, then y is not taller than x.

Here are some premises that are implicit in the meaning of 'tall':

- $\forall x \neg Txx$ (Nothing is taller than itself.)
- $\forall xy(Txy \supset \neg Tyx)$ (If object x is taller than y, then y is not taller than x.)

When judging arguments using the concept 'tall' for validity, these statements should be added to the premises because they are implicitly assumed by any use of the word 'tall'. Our argument now becomes

Tke
$\forall x \neg Txx$
$\underline{\forall xy(Txy \supset \neg Tyx)}$
$\neg Tek$

which, as you can check, is valid.

You do not need to make the implicit facts about tallness explicit in your translation of any single sentence that uses the word 'tall', but you do need to ensure that your translation is consistent with these. That is why it is important to translate "Mount Everest is the tallest mountain" as "Every mountain *except Mount Everest* is such that Mount Everest is taller than it."

There are implicit premises that recur. First, a relation can be reflexive, meaning that every object bears that relation to itself. For example, the 'is the same age as' relation is reflexive because every object is the same age as itself. For another example, the 'is at least as big as' relation is reflexive because every object is at least as big as itself. If we have a relation K that is reflexive, we can make the reflexivity explicit by adding $\forall x Kxx$ as a premise. Some relations are irreflexive, meaning that every object does not bear that relation to itself. For example, the 'is a brother of' relation is irreflexive because no one is his own brother. To express that K is an irreflexive relation, we say $\forall x \neg Kxx$.

CHAPTER 7

Applications

1. David Lewis' Version of Anselm's Ontological Argument

In David Lewis' article, "Anselm and Actuality," he tries to formalize one of Anselm's ontological arguments for the existence of God.

Saint Anselm's Ontological Argument:

(1) Whatever exists in the understanding can be conceived to exist in reality.
(2) Whatever exists in the understanding would be greater if it existed in reality than if it did not.
(3) Something exists in the understanding, than which nothing greater can be conceived.
(4) Thus, something exists in reality, than which nothing greater can be conceived.

1.1. Translation of the argument. Premise 1: Whatever exists in the understanding can be conceived to exist in reality.

$Ux = x$ is understandable.
$Wx = x$ is a world.
$Exw = x$ exists in w.
All understandable beings have some conceivable world where they exist.
$\forall x(Ux \supset \exists w(Ww \& Exw))$

Premise 2: "Whatever exists in the understanding would be greater if it existed in reality than if it did not."

$Ux = x$ is understandable.
$Wx = x$ is a world.
$Exw = x$ exists in w.
$Gxyzw =$ the greatness of x in y exceeds the greatness of z in w
For any understandable being x, and for any worlds y and z, if x exists in y but does not exist in z, then the greatness of x in y exceeds the greatness of x in z.
$\forall xyz((Ux \& Wy \& Wz \& Exy \& \neg Exz) \supset Gxyxz)$

Premise 3: "Something exists in the understanding, than which nothing greater can be conceived," which translates as, "There is an understandable being whose greatness cannot be conceived to be exceeded by the greatness of anything," which translates as, "The greatness of x is not exceeded by the greatness in any conceivable world of any being y." The remaining question about this translation is, "Which greatness of x?" There are four different versions of premise 3 that Lewis considers:

Premise 3a: There is an understandable being x, such that for no world w and being y does the greatness of y in w exceed the

greatness of x *in the actual world.*
Let $a =$ the actual world. $\exists x(Ux\&\neg\exists w\exists y(Ww\&Gywxa))$

Premise 3b: (The greatest greatness of x) There is an understandable being x and world z, such that for no world w and being y does the greatness of y in w exceed the greatness of x in z.
$\exists x\exists z(Ux\&Wz\&\neg\exists w\exists y(Ww\&Gywxz))$

Premise 3c. (Any greatness of x) There is an understandable being x, such that for no worlds z and w and being y does the greatness of y in w exceed the greatness of x in z.
$\exists x(Ux\&\neg\exists z\exists w\exists y(Ww\&Wz\&Gywxz))$

Premise 3d: (The greatness of x in the same world as the being compared to x) There is an understandable being x such that for no worlds w and being y does the greatness of y in w exceed the greatness of x in w.
$\exists x(Ux\&\neg\exists w\exists y(Ww\&Gywxw))$

Conclusion: Something exists in reality, than which nothing greater can be conceived.

There is a being x existing in the actual world such that for no world w and being y does the greatness of y in w exceed the greatness of x in the actual world.
$\exists x(Exa\&\neg\exists w\exists y(Ww\&Gywxa))$

1.2. Evaluation of validity. Which disambiguations of Anselm's argument are valid? The versions with premises 3a and 3c are valid. What's more 3c implies 3a, so you can prove that the argument with 3a is valid and then prove that 3c implies 3a, and that will necessitate that the argument version with 3c is valid.

The versions with premises 3b and 3d are invalid. Thus, we can set aside consideration of 3b and 3d as irrelevant because the corresponding arguments are invalid.

1.3. Evaluation of the Premises. To analyze the reasonability of Anselm's ontological argument, we need to discuss how plausible the premises are. Refer to Lewis' article to read his discussion of 3c. It is a bit more subtle than we can get into right now. So why should we believe 3a: "There is an understandable being x, such that for no world w and being y does the greatness of y in w exceed the greatness of x in the actual world"? In other words, why is the actual world the home of the greatest greatness?

If 3a is plausible in and of itself, that's because there is something special about our world, the actual world. One possibility is that the actual world is special because of some particular facts about our world that make this so. This isn't terribly plausible though, since we can imagine the actual world being better in almost every respect. The second possibility is that the actual world is special because there is something special about being actual. But why would actuality be a special (greatness-making property)?

Many of the features of our language about reality seem to treat reality as an indexical. An **indexical** expression is an expression whose denotation always varies according to the context of utterence. Examples include personal pronouns, demonstratives, 'here' and 'now'.

- 'I' refers to the whoever the speaker is.
- 'Here' refers to the spatial location of the speaker when he utters 'here'.
- 'Now' refers to the moment when the speaker utters 'now'.
- 'Actual' refers to the world of the speaker.

Lewis wants us to believe that alternate universes are every bit as real as our universe. Being real or actual is not a special property. Santa Claus and transparent metal exist in some worlds. They just don't inhabit the same universe that you and I inhabit. In brief, reality is an indexical notion. Relative to the world I inhabit, you exist. Relative to the world you and I exist in, Santa Claus, doesn't exist. There is an alternate universe where Santa Claus is thinking to himself, "I exist" and Santa is right. He does exist. He exists in the world where he says "I exist." But none of us exist in that world. We are mere possibilities to Santa just as Santa is to us.

The conclusion Lewis then wants us to draw is that we don't have any special property of actuality, and thus no reason to believe in 3a, and thus no reason to be motivated by Anselm's argument.

Lewis' views on actuality are not widely shared. One reason for doubting them is that our world is special in many interesting ways that would be inexplicable without there being something special about the world we inhabit. For example, many of the regularities of our world can be described in a remarkably simple mathematical form. it is compatible with all we know that all the regularities of nature as we know them will cease operation tomorrow. However, it routinely happens that the simple regularities continue to hold, and the specialness of our universe (the specialness of its laws of nature) seem to be the best explanation for the enduring regularities.

2. Models and Theories

A theory is a set of sentences. A theory in propositional logic is something like $\{R, W\&E, \neg(R \vee (E \supset \neg W))\}$. A theory in quantifier logic is something like $\{U1d\&\forall x((P1x\&\neg(x = d)) \supset \neg U1x), \exists x(P1x\&A1x), R3haf\}$.

These theories are just strings of grammatically organized symbols. They are just symbols that fit the rules for sentences (wffs with only bound variables).

The theories only have meaning when they are given an interpretation, when we settle on how the symbols relate to something.

For natural languages like English, we can think of interpretations as mappings from sentences to sets of possible worlds. For example, the sentence "No pigeons are red." is mapped to a set containing all the possible worlds that completely lack red pigeons.

For propositional logic, we think of interpretations as mappings from sentences to either TRUE or FALSE. For example, one possible interpretation of the theory $\{Q\}$ is to map the sentence Q to FALSE. Another interpretation is to map Q to TRUE. These are the only possible interpretations for the theory $\{Q\}$. There are constraints on how we can interpret the symbols. For example if we map Q to FALSE and E to TRUE, then we have to map $\neg(E \vee \neg Q)$ to FALSE. The constraints on interpretations in propositional logic are given by the truth tables for $\neg$, $\&$, $\vee$, and $\supset$. We use truth tables to tell us what all the possible interpretations of a theory are. For example, the theory $\{F\&R, \neg(M \supset \neg M), \neg R\}$ has 4 possible interpretations, one where M and R map to TRUE, one where M and R map to

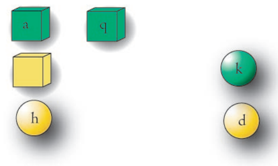

FIGURE 1. Sample Model

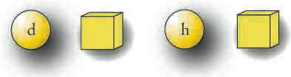

FIGURE 2. Sample Model

FALSE, one where M maps to TRUE and R maps to FALSE, and one where R maps to TRUE and M maps to FALSE.

For quantifier logic, we can think of interpretations as mappings from sentences to sets of possible models, where the models contain objects that may have properties and relations. For example, we will discuss models with objects like d, h, k, with properties 'spherical', 'cubic', 'yellow', and 'green', and with the relations 'above', and 'below'.

For the illustrations, we will stick to interpretations where the following predicates and relations are kept fixed:

- $Sx = x$ is spherical.
- $Cx = x$ is cubic.
- $Yx = x$ is yellow.
- $Gx = x$ is green.
- $Axy = x$ is above y.
- $Bxy = x$ is below y.

Here are some sentences that are true of the model pictured in figure **??**:

Gk
$Ca \& Cq$
$\forall x((Gx \vee Cx) \supset Axh)$
$\neg \exists x(Yx \& \exists y(Gy \& Axy))$

Here are some sentences that are true of the model pictured in figure **??**:

$\neg Bdh$
$\forall x(Yx \& \neg Bxd)$
$\neg \forall x(Gx \vee Axh)$
$\neg \exists x(Yx \& Axd)$

Which of the following sentences are true of the model pictured in figure **??**:

$\exists x(\neg Cx \& \neg Bkx)$
$\forall x(Yx \supset Bkx)$
$Gd \vee \neg \forall x(Cx \supset \neg Axd)$
$\neg \exists x(Sx \& \exists y Axy)$

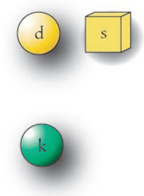

FIGURE 3. Sample Model

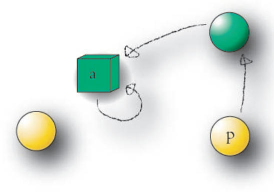

FIGURE 4. Sample Model

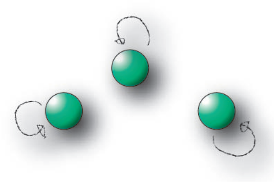

FIGURE 5. Sample Model

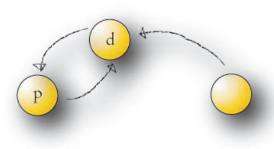

FIGURE 6. Sample Model

Suppose we add a relation Rxy. Figure **??** has an arrow going from x to y if and only if the relation R holds between x and y. Here are some sentences that are true of the model pictured above:

This relation pictured in figure **??** is interesting because it expresses a reflexive relation. A relation is reflexive if and only if every object bears that relation to itself, i.e., $\forall x Rxx$

This relation pictured in figure **??** has no objects that bear the relation to itself. We call this kind of relation irreflexive. A relation is reflexive if and only if every object does not bear that relation to itself, i.e., $\forall x \neg Rxx$. A relation can be reflexive or irreflexive or neither.

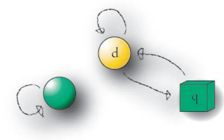

FIGURE 7. Sample Model

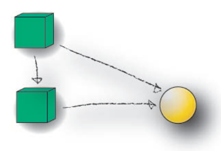

FIGURE 8. Sample Model

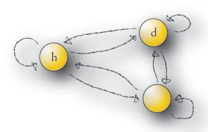

FIGURE 9. Sample Model

The model in figure **??** is neither reflexive (because $\neg Rqq$) nor irreflexive (because Rdd). This model does show a symmetric relation. A relation is symmetric if and only if whenever one object bears the relation to another, the other bears it back to the original, i.e., $\forall x(Rxy \supset Ryx)$.

Relations like the one pictured in figure **??** do not have any objects that bear a relation to some object and have that same object bear the relation back to it. Relations like these are called asymmetric. The defining property of an asymmetric relation is $\forall x(Rxy \supset \neg Ryx)$. A relation is either symmetric, asymmetric or neither.

Figure **??** illustrates a relation that is transitive. A relation is transitive if and only if whenever there is a chain of relations between one object and another, there is a (direct) relation between that object and the other, i.e., $\forall xyz((Rxy \& Ryz) \supset Rxz)$.

When a relation is reflexive, symmetric and transitive, we say that it is an equivalence relation. An equivalence relation is shown in figure **??**.

EXERCISE 17. *Draw a model with a yellow cube s, and three spheres including k. Have the model make the following sentences true:* $\{Rss, \forall x(x \neq s \supset \neg Rxx), \exists y(Sy \& Ryk), \exists z(Gz \& \forall y(Sy \supset (Rzy \& Ryz)))\}$

EXERCISE 18. *Draw a model with a yellow cube r, and three spheres. Have the model make the following sentences true:* $\{\forall x(\neg Rsx \& \neg Rxs), \exists y(Gy \& \forall x(x \neq r \supset Ryx), \exists y \exists z(Ryz \& Rzy)\}$

2.1. Rules for Reading Model Pictures. Truth

- If there is an arrow going from b to c, then Rbc is true.
- If there is not an arrow going from b to c, then Rbc is false.

Reflexivity

- If every object has a loop to itself, the relation is reflexive.
- If no object has a loop to itself, the relation is irreflexive.
- If some have a loop and others don't, it is neither.

Symmetry

- If every time there is an arrow from b to c, there is also an arrow going from c to b, the relation is symmetric.
- If every time there is an arrow from b to c, there is not an arrow going from c to b and there are no loops, the relation is anti-symmetric.
- If ([some pairs have arrows going back and forth or there are loops] and others don't have arrows going back and forth), it is neither.
- (Loops don't affect symmetry. Ignore them.)
- (Objects without arrows don't affect symmetry. Ignore them.)

Transitivity

- If every time there is an arrow from b to c, and an arrow going from c to f, there is also an arrow going directly from b to f, the relation is transitive.
- If every time there is an arrow from b to c, and an arrow going from c to f, there is not an arrow going directly from b to f, the relation is intransitive.
- If some triplets of objects fit the pattern and others don't, it is neither.
- (Objects without arrows don't affect transitivity. Ignore them.)
- (To be transitive, a pair of objects with arrows going back and forth have to also have loops on both objects.)
- (To be intransitive, there can't be any loops.)

2.2. Natural Relations. How do we check for reflexivity, symmetry and transitivity for relations that are not pictured?

For reflexivity, ask "Do the objects bear that relation to themselves?" If they have to, it is reflexive. If they can't, it is irreflexive. If they can but don't have to, it is neither. Examples:

- x is a cousin to y. Irreflexive
- x is similar to y. Reflexive
- x is bigger than y. Irreflexive
- x is a friend to y. Irreflexive
- x is located within 12 feet of y. Reflexive

For symmetry, ask "If one object bears that relation to another, does the other bear it back in return?" If it has to, it is symmetric. If it can't, it is anti-symmetric. If it can but doesn't have to, it is neither. Examples:

- x is a cousin to y. Symmetric
- x is similar to y. Symmetric
- x is bigger than y. Anti-symmetric
- x is a friend to y.
- x is located within 12 feet of y. Symmetric

For transitivity, ask "If the first object bears that relation to a second, and the second bears it to a third, does the first bear it to the third? If it has to, it is transitive. If it can't (and it is irreflexive), it is intransitive. If it can but doesn't have to, it is neither. Examples:

- x is a cousin to y.

- x is similar to y.
- x is bigger than y. Transitive
- x is a friend to y.
- x is located within 12 feet of y.

3. Elucidating Concepts

6 cheerleaders have formed what is called a pyramid, (really a triangle). Jodie is above Angela, and Conrad is below Angela. Can we infer that Conrad is below someone who is above Angela?

Aja
$\underline{Bca}$
$\exists x(Px\&(Axa\&Bcx))$

This is not valid. We have to add additional implicit premises to give our logic a chance to show that it is a valid inference. First we need to add the seemingly obvious, that Jodie, Angela and Conrad are all people.

Pj
Pa
Pc
Aja
$\underline{Bca}$
$\exists x(Px\&(Axa\&Bcx))$

We also write down a complete set of sentences that express as much as we can express about aboveness relation, A, and the belowness relation, B.

Here is what we can say about A:

- It is irreflexive: $\forall x\neg Axx$
- It is asymmetric: $\forall xy(Axy \supset \neg Ayx)$
- It is transitive: $\forall xyz((Axy\&Ayz) \supset Axz)$

Here is what we can say about B:

- It is irreflexive: $\forall x\neg Bxx$
- It is asymmetric: $\forall xy(Bxy \supset \neg Byx)$
- It is transitive: $\forall xyz((Bxy\&Byz) \supset Bxz)$

The following principle captures all we know about their connection.

- $\forall x\forall y((Axy \supset Byx)\&(Bxy \supset Ayx))$

The final argument becomes

Pj
Pa
Pc
Aja
Bca
$\forall x\neg Axx$
$\forall xy(Axy \supset \neg Ayx)$
$\forall xyz((Axy\&Ayz) \supset Axz)$
$\forall x\neg Bxx$
$\forall xy(Bxy \supset \neg Byx)$
$\forall xyz((Bxy\&Byz) \supset Bxz)$

$$\frac{\forall xy((Axy \supset Byx)\&(Bxy \supset Ayx))}{\exists x(Px\&(Axa\&Bcx))}$$

If you do the truth tree for this argument, you can verify that it is valid.

4. Family Relations

In the Computer Science department, you can learn about some programming languages called Prolog, or A-Prolog or some variant. What these languages let you do is to enter in some information in the form of sentences of first-order logic (what we have been learning all semester), and then the computer does all the inferences (truth trees). This can be used as a tool to explore creating artificial intelligence. We humans start off a computer agent with some general facts, and then the computer is able to make deductions.

Suppose we try to teach the computer about families. We want to put in some general principles about mothers, sisters, cousins, etc.

Let's start out delineating the concept of a brother. What concepts can we use to break down the meaning of being a brother?

(1) To be a brother you have to be male.
(2) To be a brother you have to have the same parents.

Let's start out assuming we have a property of being male and relationship of parent.

Let Pxy be x is a parent of y.
Let Mx be x is male.
Let Bxy be x is a brother to y.

We want to express, "x is a brother to y if and only if x is male and x has two parents w and z, and y has the same two parents w and z."

We code this up as a general statement ranging over all people x and y, $\forall x\forall y((Mx\&\exists z\exists w((Fzx\&Fzy)\&(Mwx\&Mwy))) \equiv Bxy)$.

After putting this information into the computer, we need to check to see whether it works. Suppose we tell the computer the following additional facts:

- Dustin is a parent to Gunner. Pdg
- Rachel is a parent to Gunner. Prg
- Dustin is a parent to Brytni. Pdb
- Rachel is a parent to Brytni. Prb
- Gunner is male. Mg
- Brytni is female. Fb
- Dustin is male. Md
- Rachel is female. Fr

When we ask the computer, "Is Gunner a brother of Brytni?" we are asking whether Bgb follows logically from the premises above. The computer answers "No."

When we ask the computer, "Is Brytni a brother of Dustin?" (Bbd?), the computer answers that it doesn't know. This is because it cannot infer that Brytni is, and it cannot infer that she isn't. It isn't (so far as we have programmed it) able to tell that Brytni is not male, even though it knows that Brytni is female.

We need some principle that tells us that a person can't be both male and female. This brings up some interesting questions of sexuality. Do we want to say everyone is either male or female? Do we want to say that no one can be both? For simplicity assume that there is no one of dual sex. We do this by adding $\forall x \neg(Mx \& Fx)$ where Fx means x is female. Now, when we ask the computer, "Is Brytni a brother to Dustin?" *Bbd*? the computer will correctly answer "No."

Suppose we tell the computer some new facts:

(1) Austin is male. Ma
(2) Dustin is a parent to Austin. Pda

Now, when we ask the computer, "Is Gunner a brother to Austin?" Bga The computer answers "Yes," but it ought to answer "I don't know," because they might only be half-brothers.

This is because $\forall x \forall y((Mx \& \exists z \exists w((Fzx \& Fzy) \& (Mwx \& Mwy))) \equiv Bxy)$ is poorly stated. It doesn't take account of the fact that the z and w need to be different people. Moreover, we need one to be the mother and one to be the father. We need to tell the computer what a mother is: $\forall x \forall y(Mxy \equiv (Fx \& Pxy))$ and what a father is: $\forall x \forall y(Fxy \equiv (Mx \& Pxy))$

Then our definition of brotherhood becomes:
$\forall x \forall y((Mx \& \exists z \exists w((Fzx \& Fzy) \& (Mwx \& Mwy))) \equiv Bxy)$

Now when we ask, "Is Gunner a brother to Austin?" the computer will answer, "I don't know," because it doesn't know that Rachel is Austin's mother.

Let's go ahead and tell it that Rachel is Austin's mother. Mra So now it will answer yes to "Is Gunner a brother to Austin?" and "Is Austin a brother to Gunner?"

Suppose we test it further by asking, "Who are Brytni's brothers?" The computer responds, "Gunner and Austin." Note that the computer is telling us the information so far as it knows. The computer is not telling us that those are the only people that are Brytni's brothers, but just that those are the only brothers the computer knows about.

Suppose we test it further by asking, "Who are Gunner's brothers?" The computer responds, "Gunner and Austin." But wait! It should have just said Austin. Why did it say Gunner was one of Gunner's brothers? Because we didn't tell the computer that a person can't be his own brother. So let's add it in as follows: $\forall x \neg Bxx$.

Now when we ask, "Who are Gunner's brothers?" the computer answers, "Gunner, Austin, Rachel, Dustin, Brytni." How did that happen? It happened because we got a contradiction in our premises. The definition of brother that we had still implies that Gunner is his own brother, and our new premise implies that he isn't. Thus, we have given the computer an inconsistent set of information, and from an inconsistent set, every statement whatsoever follows.

Instead of saying just $\forall x \neg Bxx$, we need to modify the definition of brotherhood as well. $\forall x \forall y((Mx \& \neg(x = y) \& \exists z \exists w((Fzx \& Fwx) \& (Mzy \& Mwy))) \equiv Bxy)$ With this change, the computer will correctly answer, "Dustin."

Suppose we ask "Is Gunner a father to Brytni?" The computer answers, "I don't know." But we know they are brother and sister, so they can't be father and daughter.

Before we add a rule ruling out siblings from being parent and child, let's test it some more. Suppose we add the fact that Phil is male. Then we ask, "Is Phil a father to Brytni?" The computer answers, "I don't know." But in this case too, we know that the answer is "No." Because we know that a person only has one father and one mother. If we can get the computer to solve this problem we can get it to fix the problem where it doesn't know that Gunner is not the father to Brytni.

So we just need to state the idea that a person only has one father. Actually, while we're at it, we might as well add something else. That everyone has a one father. Our statement then is that everyone has exactly one father:
$\forall x(\exists z Fzx \& \forall y \forall z((Fxz \& Fyz) \supset x = y))$ and the same for mother:
$\forall x(\exists z Mzx \& \forall y \forall z((Mxz \& Myz) \supset x = y))$

Let's summarize the relations and predicates we have used so far:

- Let Pxy be x is a parent of y.
- Let Mx be x is male.
- Let Fx be x is female.
- Let Mxy be x is a mother of y.
- Let Fxy be x is a father of y.
- Let Bxy be x is a brother to y.

Let's summarize the general facts we have in the computer:

- $\forall x \forall y(Mxy(Fx \& Pxy))$
- $\forall x \forall y(Fxy(Mx \& Pxy))$
- $\forall x \forall y((Mx \& \neg(x = y) \& \exists z \exists w((Fzx \& Fzy) \& (Mwx \& Mwy))) \equiv Bxy)$
- $\forall x(\exists z Mzx \& \forall y \forall z((Mxz \& Myz) \supset x = y))$
- $\forall x(\exists z Fzx \& \forall y \forall z((Fxz \& Fyz) \supset x = y))$
- $\forall x\ Pxx$

Let's summarize the particular facts we have in the computer

- Dustin is a parent to Gunner. Pdg
- Rachel is a parent to Gunner. Prg
- Dustin is a parent to Brytni. Pdb
- Rachel is a parent to Brytni. Prb
- Dustin is a parent to Austin. Pda
- Rachel is a parent to Austin. Pra
- Gunner is male. Mg
- Brytni is female. Fb
- Dustin is male. Md
- Rachel is female. Fr
- Austin is male. Ma
- Phil is male. Mp

EXERCISE 19. *How do you define a sister relation? A grandmother relation? A cousin relation?*

Let's summarize the additional relations and predicates:

- Let Pxy be x is a parent of y.
- Let Sxy be x is a sister to y.
- Let Bxy be x is a brother to y.
- Let Sxy be x is a sister to y.
- Let Gxy be x is a grandparent to y.
- Let $G^f xy$ be x is a grandfather to y.
- Let $G^m xy$ be x is a grandmother to y.
- Let Cxy be x is a cousin to y.
- Let Axy be x is an aunt to y.
- Let Uxy be x is an uncle to y.

- Let Jxy be x is a nephew to y.
- Let Nxy be x is a niece to y.
- Let Oxy be x is a great-grandparent to y.

(Treat expressions with superscripts, e.g. G^f, just like a relation letter. This allows us to more easily remember what the relation symbols stand for.)

Let's summarize the general facts we already have in the computer:

- $\forall xy(Mxy \equiv (Fx\&Pxy))$ definition of mother
- $\forall xy(Fxy \equiv (Mx\&Pxy))$ definition of father
- $\forall xy((Mx\&\neg(x = y)\&\exists z(Fzx\&Fzy)\&\exists w(Mwx\&Mwy)) \equiv Bxy)$ definition of brother
- $\forall xy((Fx\&\neg(x = y)\&\exists z(Fzx\&Fzy)\&\exists w(Mwx\&Mwy)) \equiv Sxy)$ definition of sister
- $\forall x\neg Bxx$ No one is his or her own brother.
- $\forall x\neg Sxx$ No one is his or her own sister.
- $\forall x\exists zMzx\&\forall xyz((Mxz\&Myz) \supset x = y)$ Everyone has exactly 1 mother
- $\forall x\exists zFzx\&\forall xyz((Fxz\&Fyz) \supset x = y)$ Everyone has exactly 1 father
- $\forall x\neg(Mx\&Fx)$ No one is both male and female.
- $\forall x(Mx \vee Fx)$ Everyone is either male or female.
- $\forall x\ Pxx$ No one is his or her own parent.

How do you tell the computer enough so that it can answer questions about grandmothers and grandfathers?

- $\forall xy(Gxy \equiv \exists z(Pxz\&Pzy))$ definition of grandparent
- $\forall xy(G^m xy \equiv (Gxy\&Fx))$ definition of grandmother
- $\forall xy(G^f xy \equiv (Gxy\&Mx))$ definition of grandfather
- $\forall x\neg Gxx$ no one is his or her own grandparent

How do you tell the computer enough so that it can answer questions about cousins?

- $\forall x\forall y(Cxy \equiv (\exists z(G^m zx\&G^m zy)\&\exists w(G^f wx\&G^f wy)\&(x \neq y)\&\neg(Bxy \vee Sxy)))$ definition of cousin

How do you tell the computer enough so that it can answer questions about half-sisters and half-brothers?

- $\forall x\forall y((Mx\&(\exists z(Fzx\&Fwx) \equiv \neg\exists w(Mzy\&Mwy))) \equiv B^{1/2}xy)$ definition of half-brother
- $\forall x\forall y((Fx\&(\exists z(Fzx\&Fwx) \equiv \neg\exists w(Mzy\&Mwy))) \equiv S^{1/2}xy)$ definition of half-sister

5. Foundations of Arithmetic

We can express fundamental axioms of arithmetic with first-order logic. First order logic is the full quantifier logic with identity and with function symbols. A function is a mapping from some number of terms to a single term. For example, suppose $m(x)$ is the function that takes any person x as input and outputs the mother of x. So if j represents Jocasta and o represents Oedipus, then $m(o) = j$. (Remember your Greek mythology to see that this is true.) Sometimes we write functions in other ways. For example, the *plus* function is defined so that $plus(x, y)$ outputs the sum of x and y, and we often write this function as $x + y$.

Let o be the constant (which we will eventually interpret as the number zero) and let $s(x)$ be a function which we call the successor of x. We will interpret the successor of x as the number that follows x in counting the ordinary way. For example, 15 is the successor of 14.

(1) $\forall x \forall y((s(x) = s(y)) \supset (x = y))$
(2) $\forall x(0 \neq s(x))$
(3) $\forall x(x \neq 0 \supset \exists y(x = s(y)))$
(4) $\forall x(x + 0 = x)$
(5) $\forall x \forall y(x + s(y) = s(x + y))$
(6) $\forall x(x \times 0 = 0)$
(7) $\forall x \forall y(x \times s(y) = (x + y) + x)$

With just these axioms, you can prove a lot of arithmetic.

Gödel's 1st Incompleteness Theorem: Kurt Gödel proved that with any consistent, finite set of axioms that is powerful enough to do arithmetic, there are always some arithmetical truths that cannot be proved from the axioms.