

Philosophy 109, Modern Logic, Queens College
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Truth tables for arguments, §6.4

I. Compare the following arguments:

A)
 1. If God exists then every effect has a cause.
 2. God exists.
 \therefore Every effect has a cause.

B)
 1. If God exists then every effect has a cause.
 2. Every effect has a cause.
 \therefore God exists.

A) is valid, and has the following form:

$P \supset Q$
 P
 $/ Q$

This form is known as Modus Ponens.

Note that we write the premises on sequential lines, and the conclusion on the same line as the final premise, following a single slash.

B) is invalid, and has the following form:

$P \supset Q$
 Q
 $/ P$

This is known as the Fallacy of Affirming the Consequent.

Recall: In a **valid argument**, if the premises are true then the conclusion must be true.

Note, it says nothing about what happens if the premises are false.

An **invalid argument** is one in which it is possible for true premises to yield a false conclusion.

By focusing on valid arguments, we can make sure that if all our premises are true, so must our conclusions be.

II. A method for determining if an argument is valid.

Step 1: Line up premises and conclusion horizontally, separating premises with a single slash and separating the premises from the conclusion with a double slash.

Step 2: Construct truth tables for each premise and the conclusion, using consistent assignments to component variables.

Step 3: Look for a counter-example: a row in which all premises are true and the conclusion is false.

If there is a counter-example, the argument is invalid.

If there is no counterexample, the argument is valid.

Valid argument:

P	\supset	Q	/	P	//	Q
T	T	T		T		T
T	F	F		T		F

F	T	T		F		T
F	T	F		F		F

Note: On no line are the premises true and the conclusion false. There is no counter-example

Invalid argument:

P	\supset	Q	/	Q	//	P
T	T	T		T		T
T	F	F		F		T
F	T	T		T		F
F	T	F		F		F

This third row is the *counterexample*:
The argument is invalid when P is F and Q is

Consider: Is this a valid argument?

1. $P \supset (Q \supset P)$
2. $\sim P$ / Q

P	\supset	(Q	\supset	P)	/	$\sim P$	//	Q
T	T	T	T	T		F		T
T	T	F	T	T		F		F
F	T	T	F	F		T		T
F	T	F	T	F		T		F

Row 4 is a counter-example.

The argument is shown invalid when P is false and Q is false.

III. Exercises. Determine whether each argument is valid. If invalid, specify the counter-example.

- 1)
 $A \supset B$
 $\sim A$ / $\sim B$

- 2)
 $C \vee D$
 $\sim D$ / C

- 3)
 $E \equiv F$ / $\sim E \vee F$

- 4)
 $G \cdot H$
 $H \supset I$ / $\sim(G \cdot I)$

- 5)
 $J \supset K$
 $K \supset \sim J$
 $\sim J \supset K$ / $J \vee \sim K$

IV. Solutions

1. Invalid, when A is false, and B is true.
2. Valid
3. Valid
4. Invalid, when G, H, and I are all true.
5. Invalid, when J is false and K is true.