The Language Revolution
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Class #4
Frege's Projects

Today

- Frege's logic and its legacy
  - Syntax
  - Semantics
  - Pragmatics
- Back to Locke and Meinong and Mill
  - A group activity
- Frege's three principles

Gottlob Frege

⇒ 1848-1925

⇒ Influence on every area of contemporary philosophy: logic, mathematics, language, science, mind, metaphysics, epistemology

⇒ His work all derives from an attempt to provide a foundation for mathematics.

⇒ The language revolution is a side project.

an attempt to work out the bugs in the formal language he devised to proved a syntactic, algorithmic definition of logical consequence

Nineteenth Century Developments in Mathematics

some oddities

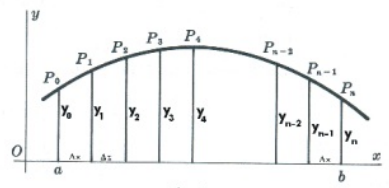
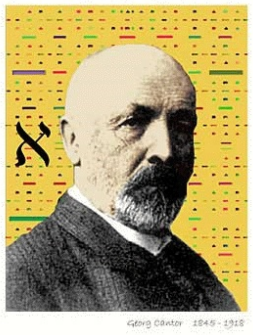


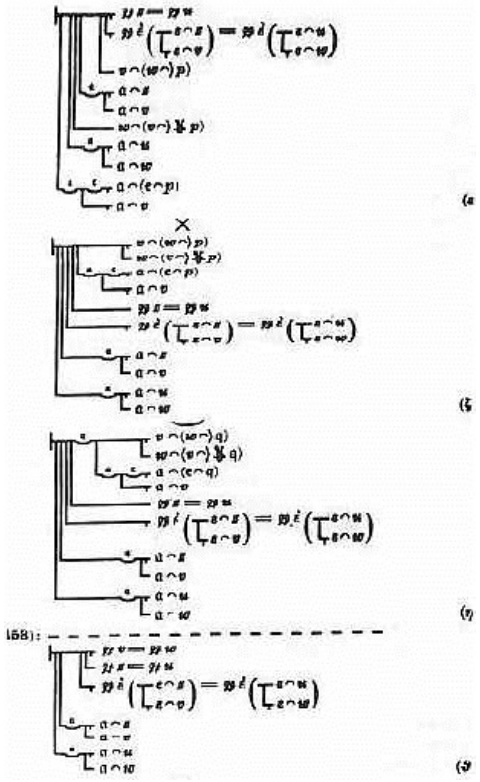
Fig. 5

- The calculus of infinitesimals
- Cantor's proof that there are different sizes of infinity
- Non-Euclidean geometries
  - Euclid's parallel postulate (via Playfair's postulate): given a line, and a point outside that line, there is one and only line which passes through the point parallel to the given line.
  - no parallel lines: the geometry of spheres
  - more than one parallel line: hyperbolic geometry
  - Hyperbolic geometry is not only to be consistent, it's the correct geometry for space-time.
- Mathematicians and philosophers began to think more carefully about the notion of logical consequence: what follows from what?



Frege's Logicism

- *Logicism*: mathematics is just logic in disguise.
- Logic is obvious and uncontroversial.
- If mathematics were just logic in disguise, then it too would be uncontroversial.
- To show that mathematics is just logic, Frege first had to formalize the notion of a proof.
- *Begriffsschrift*, or concept-writing



Logical Truth and Language

- Using the formal language in the *Begriffsschrift*, Frege was able to refine the notion of logical truth.
- Logical truths depend merely on language.
  - all bachelors are unmarried
  - if p then p
  - perhaps '2+2=4'
- Hume had called such truths relations of ideas.
  - They follow from the principle of non- contradiction.
- Kant declared such truths *a priori*.
- Frege provided a formal method for characterizing these truths.

Frege and the Language Revolution

- Encouraged by the clarity and precision of Frege's new formal language, philosophers developed hopes of using it as canonical in all proper scientific discourse.
- 1879: *Begriffsschrift*.
- 1884: *Grundlagen* (*Foundations of Arithmetic*), defending logicism
- 1892: "Über Sinn und Bedeutung" ("On Sense and Reference") and "Über Begriff und Gegenstand" ("On Concept and Object")
  - the seminal work of the revolution
- The *Grundgesetze*, in two (of three planned) volumes (1893 and 1903), did some of the technical work promised in the *Grundlagen*.
- 1903: Russell's paradox
  - the set of all sets which do not include themselves
  - Frege's project never recovered.
- Frege's "The Thought" (for Thursday) was published in 1918.

Frege's Microscope

I believe I can make the relationship of my *Begriffsschrift* to ordinary language clearest if I compare it to that of the microscope to the eye. The latter, due to the range of its applicability, due to the flexibility with which it is able to adapt to the most diverse circumstances, has a great superiority over the microscope. Considered as an optical instrument, it admittedly reveals many imperfections, which usually remain unnoticed only because of its intimate connection with mental life. But as soon as scientific purposes place great demands on sharpness of resolution, the eye turns out to be inadequate. The microscope, on the other hand, is perfectly suited for such purposes... (Frege, Preface to *Begriffsschrift*)

Frege and Leibniz on *A Priori* Knowledge

- Frege: “[W]e divide all truths that require justification into two kinds, those whose proof can be given purely logically and those whose proof must be grounded on empirical facts. But there is no inconsistency in a proposition belonging to the first kind and yet being such that it can never be apprehended by a human mind without the operation of the senses.”
- Leibniz: “Although the senses are necessary for all our actual knowledge, they are not sufficient to provide it all, since they never give us anything but instances, that is particular or singular truths. But however many instances confirm a general truth, they do not suffice to establish its universal necessity; for it does not follow that what has happened will always happen in the same way” (Leibniz, *New Essays,* 49).
- Poverty of the evidence claim
  - Chomsky

Frege's Legacy in Language

- Formal theories
  - Contemporary linguistic theories rely on formal theories of syntax.
  - Some philosophers and linguists attempt to construct semantic theories
- A short step to the distinction between syntax and semantics
  - Syntax: the formal properties of languages
  - Semantics: the content of language

What We Can Do With Formal Systems

1. Construct a language
2. State some axioms, or basic principles, for a theory
3. Provide rules of inference, to derive other theorems
4. Interpret, or model, the theory

The MIU system

from Hofstadter, *Gödel, Escher, Bach*

- Any string of Ms Is and Us is a string of the MIU system.
  - MIU, UMI, and MMMUMUUUMUMMU are all strings.
- Some privileged strings are theorems.
- Similarly, any declarative sentence in English corresponds to the strings of a formal system.
  - In English, we may be interested in only the true sentences.
  - In the MIU system, we will only be interested in theorems.

Axioms and Theorems

- An axiom is an assumption.
- A theorem is any string which is either an axiom, or follows from the axioms by using some combination of the rules of inference.
- The MIU system takes only one axiom: MI.

Rules of Inference

Axiom: MI

R1. If a string ends in I you can add U.

R2. From Mx, you can infer Mxx.

- That is, you can repeat whatever follows an M.

R3. If III appears in that order, then you can replace the three Is with a U

R4. UU can be dropped from any theorem.

Rules of Inference and Theorems

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Some theorems of MIU

1. MI	Axiom
2. MIU	From Step 1 and R1
3. MII	1, R2
4. MIIII	3, R2
5. MIU	4, R3
6. MUI	4, R3
7. MIIIIIIII	4, R2
8. MIUUI	7, R3
9. MII	8, R4

Derive MIIIII

(That's five 'I's)

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A Challenge for Later: Derive ‘MU’

- For help, see Hofstadter 259-261.
- Do not spend too much time on this puzzle without consulting Hofstadter, who provides helpful hints!

Syntax without Semantics

- In the MU system, there is no indication what any of the theorems or strings mean.
- Natural language has syntactic elements: grammaticality
- Linguists tend to focus on syntax.
- Philosophers of language are less interested in syntax than they are in semantics and pragmatics.
- Martinich has an introduction to syntax and semantics in §VII - §VIII.

Speaking of Martinich

Semantic Theories and Truth Theories

- ‘Semantics’ can be used to refer to two different kinds of theories
  - a semantic theory
  - a theory of truth
- A semantic theory should yield an infinite number of theorems, one for each sentence of the language.
  - Are there infinitely many sentences of English?
  - How many tweets? (XKCD: ~2×10^{46})
- A metalanguage and an object language can be the same.
  - S1 ‘snow is white’ means-in-English that snow is white
  - S2 ‘grass is green’ means-in-English that grass is green
- The metalanguage can differ from the object language.
  - S3 ‘la nieve es blanca’ means-in-Spanish that snow is white
  - S4 ‘la hierba es verde’ means-in-Spanish that grass is green

Truth Theories

- Start with a minimal condition, Convention T
  'p' is true iff x
- Some T-theorems
  T1 'snow is white' is true if and only if snow is white
  T2 'grass is green' is true if and only if grass is green
  T3 '2+2=4' is true if and only if 2+2=4
  T4 'Barack Obama is president' is true if and only if the husband of Michelle Obama and father of Sasha Obama and Malia Obama is head of the executive branch of the United States of America.
  T5 'El gato está en el alfombrilla' is true iff the cat is on the mat.

Pragmatics

- In addition to syntax, semantics, and truth, some philosophers of language are also interested in the difference between what is said and what is communicated.
- The study of what gets communicated is called pragmatics.
- Pragmatics is the study of what kinds of acts can be performed with language, in addition to the meaning of one's words.
- Frege makes some brief comments on pragmatics in "The Thought," mainly to show their independence from meaning.
- We'll take on pragmatics, a bit, at the end of the course.

Contrastive Stress

- Humpty Dumpty was sitting with his legs crossed, like a Turk, on the top of a high wall - such a narrow one that Alice quite wondered how he could keep his balance - and, as his eyes were steadily fixed in the opposite direction, and he didn't take the least notice of her, she thought he must be a stuffed figure after all.
- "And how exactly like an egg he is!" she said aloud, standing with her hands ready to catch him, for she was every moment expecting him to fall.
- "It's *very* provoking," Humpty Dumpty said after a long silence, looking away from Alice as he spoke, "to be called an egg - *very*!"
- "I said you *looked* like an egg, Sir," Alice gently explained. "And some eggs are very pretty, you know," she added, hoping to turn her remark into a sort of a compliment.

In Three Groups: Locke, Meinong, Mill

How does each philosopher understand each sentence? Consider truth and falsity, and meanings of the terms.

1. The Empire State Building is tall, with blue lights.
2. The king of America has a lot of power.
3. Kanye West will run for president in 2020.

Take 7 minutes.

Frege's Three Principles

- Logicism and the *Grundlagen*
- Three guidelines:
  - FG1. Always to separate sharply the psychological from the logical, the subjective from the objective;
  - FG2. Never to ask for the meaning of a word in isolation, but only in the context of a proposition;
  - FG3. Never to lose sight of the distinction between concept and object (Frege, *Grundlagen x).*
- On FG1
  - In contrast to the Moderns and the nineteenth-century idealists, Frege wants a logical, objective theory of mathematics and language.
  - Not a psychological account like Locke's.
  - Not a subsistence account, like Meinong's, but that's maybe closer.

The Context Principle

FG2. Never to ask for the meaning of a word in isolation, but only in the context of a proposition

- ‘Theaetetus’ itself has no meaning independently of how we use that term in an assertion.
- There might be many Theaetetuses.
- Even a baby’s use of the single term ‘Mama’, which might be taken as a counter-example to FG2, is best taken as an assertion, ‘There is my mother’, rather than as a mere label.
- If we take terms, like ‘two’, as labels, outside of an assertion, we end up looking for the referents of such terms.
  - Since there are no twos in the world, we end up thinking that ‘two’ refers to my idea of a two.
  - We end up believing that the referents of my terms are ideas, and we are back with Locke, Berkeley, Hume, and Kant stuck in our phenomenal worlds.
  - Giving up FG2 leads back to idealism.

Context and Compositionality

- FG2 should not be interpreted so as to contradict the claim that (human) languages are essentially compositional.
- Compositionality requires that the meanings of the whole proposition are constructed out of the meanings of the parts, and that the truth of the proposition depends on the truth of its parts.
- The context principle demands that the meanings of the parts are somehow dependent on the meanings of the whole.
- Compositionality is a fundamental principle of both logic and formal theories of language.
- Compositionality and the context principle are in tension, but they can be seen to work together.

Frege's Third Principle

FG3. Never to lose sight of the distinction between concept and object.

- Frege and Mill disagree concerning FG3.
  - Mill thinks of 'blue' as the name of a thing, a general name, rather than the name of a concept.
  - Frege spends a lot of the *Grundlagen* attacking Mill's empiricist philosophy of mathematics.
  - FG3 is a central part of both Frege's attack on Mill and his positive account of numbers.

Frege Against Idealism

idealism leads to a regressive absurdity

- Every idea requires a thinker.
- But the principles underlying idealism allow the inference only to further ideas.
- So those principles that I can know only my own ideas must be flawed.
  - If we call what happens in our consciousness idea, then we really experience only ideas but not their causes (Frege, “The Thought,” 304).
  - It is inconceivable that I should be boxed into myself in this way to infinity (305).
  - If man could not think and could not take something of which he was not the bearer as the object of his thought, he would have an inner world but no outer world. But may this not be based on a mistake? (306)
  - Not everything that can be the object of my understanding is an idea (307).

A Secondary Fregean Argument Against Idealism

- If idealism were true, then psychology would be the most fundamental science.
- But, psychology is subordinate to mathematics.
- So, idealism is false.
  - "Not everything is an idea. Otherwise psychology would contain all the sciences within it or at least it would be the highest judge over all the sciences. Otherwise psychology would rule over logic and mathematics. But nothing would be a greater misunderstanding of mathematics than its subordination to psychology. Neither logic nor mathematics has the task of investigating minds and the contents of consciousness whose bearer is a single person" (Frege, "The Thought" 308).

A Tertiary Argument

Psychology can tell us about how our beliefs are formed, but not whether they are true.

"Error and superstition have causes just as much as genuine knowledge. The assertion both of what is false and of what is true takes place in accordance with psychological laws. A derivation from these and an explanation of a mental process that terminates in an assertion can never take the place of a proof of what is asserted" (Frege, "The Thought" 290).

For Thursday

- We have said enough about idealism.
- Let's go to Frege's positive proposal about the objects of language in "The Thought".
- Sarah will lead us!