I. Intensions, Extensions, Sense and Reference

We saw that ‘creature with a heart’ and ‘creature with kidneys’ have the same extension while differing in intension. The differences between extension and intension are, approximately, differences between the senses of the phrases and their references. Frege’s central claim as the founder of twentieth-century philosophy of language is his view that terms, both singular terms and whole propositions, have both sense and reference. Frege presents three arguments that terms of our language have both sense and reference. These arguments arise from considerations involving three puzzles.

FP1 Identity statements
FP2 Failure of presupposition/Empty reference
FP3 Opaque contexts

II. FP1: Identity Statements

‘Hesperus’ is the Greek name for the evening star.
‘Phosphorus’ is the Greek name for the morning star.
Compare HH with HP.

HH Hesperus is Hesperus
HP Hesperus is Phosphorus.

HH seems uninformative, knowable a priori, an analytic truth of language or logic, and necessarily true.
HP seems informative and knowable only a posteriori, a synthetic, contingent, empirical truth.
Given that ‘Hesperus’ and ‘Phosphorus’ refer to the same thing, how can there be such a difference?
This question is sometimes called Frege’s puzzle.
Frege uses it to motivate the distinction between sense and reference at the beginning of the article.
It is also sometimes called the problem of cognitive content.
HH and HP may differ in cognitive content.
The problem is to show how that can be.

Frege’s solution to FP1 is that names, in fact all singular terms, have both sense and reference.

It is natural, now, to think of there being connected with a sign (name, combination of words, letter), besides that to which the sign refers, which may be called the reference of the sign, also what I should like to call the sense of the sign, wherein the mode of presentation is contained (37).

‘Hesperus’ and ‘Phosphorus’ have the same reference.
But, they have different senses.
Senses are the intensions for which Frege argued in “The Thought”.
Each of us grasps a term under what Frege calls a mode of presentation.
Different people associate different senses with a given name or predicate. For Frege, senses are conjunctive sets of properties, those which we associate with a given term.

References of names are the objects that those terms designate. References of predicates can be either properties (if one is an intensionalist) or sets of objects that fall under the concept expressed in the predicate (if one is an extensionalist). So, the reference of ‘is a kitten’ will be either the property of being a kitten or just the set of kittens.

When we ask for the meaning of a term, our question may be ambiguous. We could be asking for the sense of the term. We could be asking for the reference of the term.

A proper name (word, sign, sign combination, expression) expresses its sense, stands for or designates its reference. By means of a sign we express its sense and designate its reference (40).

Recall the Lauben case from ‘The Thought’. Frege claims that Garner and Peter don’t speak the same language. Garner and Peter are referring to the same person, to Dr. Lauben, but under different modes of presentation. That is, they associate a different sense to the term ‘Dr. Lauben’, which has the same referent.

Note Frege’s connection between sense and reference.

The sense of a proper name is grasped by everybody who is sufficiently familiar with the language or totality of designations to which it belongs; but this serves to illuminate only a single aspect of the reference, supposing it to have one. Comprehensive knowledge of the reference would require us to be able to say immediately whether any given sense belongs to it. To such knowledge we never attain (37).

If we knew everything about a referent, then any mode of presentation of that referent would be subsumed under the sense which we associate with it. If I knew every fact about Barack Obama, then I could know anything you would say about him. In a logically perfect language, different senses for proper names might be avoided. But, in our language, we use names to refer to objects about which we do not know everything.

Frege makes two presumptions here. The first presumption is that the sense of a term determines its reference. In fact, one way to specify the meaning of Fregean senses is via reference: the sense of an expression is that which determines its reference. The second presumption is that my thoughts determine the sense of a sentence, what I mean. The sense of a term is completely transparent to the speaker. These presumptions will lead to trouble, later. For now, I will discuss briefly one problem arising from Frege’s characterization of the sense of a proper name as a conjunctive set of properties which form the mode of presentation of an object for a speaker.
The sense of ‘Aristotle’ could be, for one person, SA1.

SA1 teacher of Alexander the Great, student of Plato, born in Stagira, author of *Posterior Analytics*

Another person could associate a different sense with ‘Aristotle, say SA2.

SA2 Author of *Metaphysics*, guy who believed in four causes

A problem with taking senses to be sets of properties is that if SA1 is the sense of ‘Aristotle’, then PA is an analytic truth.

PA Aristotle was a student of Plato

But if SA2 is taken as the sense of Aristotle, then PA turns out to be synthetic. Having the analyticity of a sentence vary by person seems counter-intuitive. We will pursue this worry about the Fregean analysis of sense in due course. For now, let’s just be clear that the sense of a term is how we apprehend it. Frege’s distinction allows him to explain the difference in cognitive content between HH and HP. The terms in HH and HP differ in their senses even though they have the same references. Thus, identity statements, Frege says, relate senses (which are linguistic) rather than objects or references (which are, generally, non-linguistic).

III. FP2: Failure of Presupposition

Consider SC.

SC Santa Claus lives at the North Pole.

If the meaning of ‘Santa Claus’ were merely its reference, then SC is problematic. Natural languages are compositional. The meaning of the whole proposition is composed of the meanings of its parts. But if SC depends for its meaningfulness that its parts all have meanings, then SC may be problematic. For, one of its parts (‘Santa Claus’) lacks reference. This puzzle is sometimes called the problem of empty reference.

Locke avoids problems of empty reference by making ‘Santa Claus’ refer to my idea of Santa Claus. He thus avoids ascribing meaninglessness to SC. But, he makes SC and RM both refer to my ideas.

RM Russell Marcus lives at 23 Fountain Street

Against Locke’s view, it seems that RM is true and about the world, rather than my ideas. It says something true about a real person. SC is false, strictly speaking, rather than meaningless, because there is no person to whom it refers. That is, both SC and RM refer to persons, not to ideas.
Frege’s solution, to separate sense and reference, allows the compositionality of meaning even when some terms lack reference.

SC can be meaningful (i.e. it has a sense) because ‘Santa Claus’ has sense, even though it lacks reference.

Similar analyses can be given for descriptions which pick out a unique individual.

PH The president of Hamilton College wears bangs.

With referring, or definite, descriptions, the claim that they have sense is most plausible. On the other hand, as Mill pointed out, some terms really seem merely denotative, and to lack connotation.

Mill used the example of ‘Dartmouth’.

We will return to the question of whether names have sense.

Independently of the sense/reference distinction, one might think that SC entails or implies that there is a Santa Claus.

Frege argues that uses of such terms do not carry with them such a presumption.

The sense/reference distinction thus solves the problem of empty reference.

Such a problem arises only because of the sloppiness of natural language.

A logically perfect language (Begriffsschrift) should satisfy the conditions, that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact designate an object, and that no new sign shall be introduced as a proper name without being secured a reference. The logic books contain warnings against logical mistakes arising from the ambiguity of expressions. I regard as no less pertinent a warning against apparent proper names having no reference... ‘The will of the people’ can serve as an example; for it is easy to establish that there is at any rate no generally accepted reference for this expression. It is therefore by no means unimportant to eliminate the source of these mistakes, at least in science, once and for all (48).

IV. Propositions

Before we get to the argument from opaque contexts, we should look at what Frege thinks of the nature of propositions, given the sense/reference distinction.

Propositions are senses (meanings) of sentences.

‘Marina is a kitten’ means (i.e. has the sense) that Marina is a kitten.

So, let’s consider the proposition that Marina is a kitten.

We know from ‘The Thought’ that propositions are abstract, objective, mind- and language-independent entities that exist in a third ontological realm, neither private, nor physical.

We know from compositionality that propositions are structured entities built out of their parts.

The sentence MK contains a subject, ‘Marina’, and a predicate, ‘is a kitten’.

MK Marina is a kitten.

Let’s consider the components of the corresponding proposition.

It could contain the sense of ‘Marina’, the reference of ‘Marina’ (i.e. Marina herself), or something else.
Similarly, it could contain the sense of ‘is a kitten’, the reference of ‘is a kitten’ (i.e. the set of kittens), or something else.

If propositions contained the referents of their component terms, then the proposition that the morning star is the evening star would be the same proposition as that the morning star is the morning star. Since we know that those two propositions have different senses, Frege concludes, the sense of the proposition must not merely include the referents of its components. Frege believes that propositions are composed of the sense of the subject, or the subject under a mode of presentation, and the sense of the predicate, or the property under a mode of presentation.

For the proposition expressed by MK, Marina instantiates the sense (or individual concept) of the subject part of the proposition, and the property of being a kitten is an instance of the sense of the predicate, of the concept of kittenhood. If both concepts (of Marina and of kittenhood) are instantiated, then the proposition that Marina is a kitten will be true or false depending on whether or not Marina is a kitten. Otherwise, the proposition has no truth value.

Thus, in addition to the inscription or utterance of the sentence, for a person to understand an assertion, Frege posits not only objects (e.g. Marina, kittens) but also properties (e.g. being a kitten) and individual concepts (e.g. of Marina). Someone might worry that Frege is multiplying entities beyond necessity. Do we really need kittens, propositions, properties of being a kitten, and concepts of particular kittens in order to explain our understanding of MK? Frege argues that such posits are not excessive. Senses are available to us; we can think about them (or apprehend them). The property of being a kitten (or the set of kittens) is the referent of ‘x is a kitten’. But, we think about that property, or that set, only under a mode of presentation to us, i.e. its sense. The multiplication of entities is exactly how we solved the identity and empty reference problems.

Note that in addition to sense and reference, terms also evoke images in the minds of people who use them.

The same sense is not always connected, even in the same man, with the same idea. The idea is subjective: one man’s idea is not that of another. There result, as a matter of course, a variety of differences in the ideas associated with the same sense. A painter, a horseman, and a zoologist will probably connect different ideas with the name ‘Bucephalus.’ This constitutes an essential distinction between the idea and the sign’s sense, which may be the common property of many and therefore is not a part or a mode of the individual mind (39).

The imagery, being private, is no part of the sense of the term. Imagery is psychological, whereas both senses and references are objective.

V. Compositionality, Sense and Reference

So far, we have mostly been considering the senses and references of singular terms: names and descriptions which pick out individuals. I also mentioned the sense and reference of predicates, like ‘is a kitten’. Like names and definite descriptions, predicates are sub-sentential units.
The principle of compositionality for senses is that the sense of a whole expression is determined by the senses of its (often sub-sentential) parts. Some sentences are complex. Some complex sentences, like DC, are made of multiple sentences.

\[ DC \quad \text{The dog barks and the cat purrs.} \]

The parts of DC include DC1 and DC2.

\[ DC1 \quad \text{The dog barks.} \]
\[ DC2 \quad \text{The cat purrs.} \]

DC1 and DC2 are sentences themselves. By compositionality, the sense of the complex DC is determined by the senses of its parts, including the sense of ‘and’.

The meaning of DC is determined by the meanings of DC1 and DC2.

We have already seen that the proposition is the sense of the sentence.

We have also seen that senses of simple sentences are composed of senses of their sub-sentential parts. Now, we see that the sense of a complex sentence can be composed of the senses of fully sentential parts.

If the principle of compositionality were to apply also to references, then the reference of a sentence would be determined by the references of its parts.

The question is whether just as singular terms have sense and reference, so sentences will also have sense and reference.

You might believe that sentences have both sense and reference on the basis of an argument from symmetry.

The argument is that parts of simple sentences (singular terms) have both sense and reference, so parts of complex sentences (simple sentences) have both sense and reference.

On the other hand, perhaps a sentence has only a sense, and no reference.

To show that sentences, like names and predicates, have both sense and reference, Frege argues that our interest in language is subordinate to our interest in truth.

There has to be a role for truth value in analyzing a language.

If it were a question only of the sense of the sentence, the thought, it would be unnecessary to bother with the reference of a part of the sentence; only the sense, not the reference, of the part is relevant to the sense of the whole sentence. The thought remains the same whether ‘Odysseus’ has reference or not. The fact that we concern ourselves at all about the reference of a part of the sentence indicates that we generally recognize and expect a reference for the sentence itself (42).

Now, compositionality carries with it a commitment to Leibniz’s law, substitutivity \textit{salva veritate}.

\[ LL \quad \textit{Eadem sunt, quae sibi mutuo substitui possunt, salva veritate.} \quad (\text{Those things are identical of which one can be substituted for the other without loss of truth.}) \]

Substitutivity is just a basic principle of identity.

We use LL in symbolic logic.
LL1 describes the formal rule.

\[
\text{LL1} \quad \mathcal{T}a \\
\quad a = b \quad / \quad \mathcal{T}b
\]

LL2 is an instance of Leibniz’s law.

\[
\text{LL2} \\
2 + 2 = 4 \\
4 = 9 - 5 \\
\text{Thus, } 2 + 2 = 9 - 5
\]

When substituting expressions with different senses but with the same reference, the sense of the sentence changes.
But, the reference of the sentence should remain the same.
Frege argues that in ordinary cases, the truth-value will remain the same under such substitution.
So, we’ll call the referents of sentences their truth values.
If the referents of sentences are their truth values, then all true sentences refer to the same thing, which Frege calls the True.
All false sentences refer to the False.

It is the striving for truth that drives us always to advance from the sense to the reference. We have seen that he reference of a sentence may always be sought, whenever the reference of its components is involved; and that this is the case when and only when we are inquiring after the truth value. We are therefore driving into accepting the truth value of a sentence as constituting its reference. By the truth value of a sentence I understand the circumstances that it is true or false. There are no further truth values. For brevity I call the one the True, the other the False (42).

VI. FP3: Opaque Contexts

We have come to Frege’s third motivation for the sense-reference distinction.
Having all sentences refer to the True or the False is awkward, and arises mostly from the need to say something about the referents of sentences.
Still, there are considerations of indirect discourse that support Frege’s system.

If words are used in the ordinary way, what one intends to speak of is their reference. It can also happen, however, that one wishes to talk about the words themselves or their sense (38).

In cases of indirect discourse, a term may be taken to refer to its own sense.

\[
\text{LS} \\
\text{Lois Lane believes that Superman can fly.}
\]

LS contains as a part the proposition that Superman can fly.
Given compositionality, the reference of the that-clause should be the True.
But, if ‘Superman can fly’ referred to the True, then we should be able to substitute any truth and retain the truth value of the whole.
What if we replace ‘Superman can fly’ in LS with ‘Hamilton College is in Clinton NY’?
LH Lois Lane believes that Hamilton College is in Clinton NY.

We’ve replaced a truth in LS with a truth to form LH.
But Lois Lane, presumably, has no opinion on the location of Hamilton College.
Similarly, if we replace ‘Superman’ in LS with ‘Clark Kent’, we get a shift in truth value.

LC Lois Lane believes that Clark Kent can fly.

LH and LC are false.
Substitutivity fails.

Frege concludes that within such contexts, which are called opaque, references of sentences and singular terms are not their ordinary references, but their senses.
In opaque contexts, discourse is indirect.
We already need indirect discourse in order to talk about the senses of words themselves.

In order to speak of the sense of an expression ‘A’ one may simply use the phrase ‘the sense of the expression “A”’. In reported speech one talks about the sense, e.g., of another person’s remarks. It is quite clear that in this way of speaking words do not have their customary reference but designate what is usually their sense. In order to have a short expression, we will say: In reported speech, words are used indirectly or have their indirect reference (38).

Similarly, when we talk about what someone else believe (or hopes or desires), we are talking about not just the true and the false, but about how they believe it.

The situation is similar in the case of expressions such as ‘to be pleased,’ ‘to regret,’ ‘to approve,’ ‘to blame,’ ‘to hope,’ ‘to fear.’ If, toward the end of the battle of Waterloo, Wellington was glad that the Prussians were coming, the basis for his joy was a conviction. Had he been deceived, he would have been no less pleased so long as his illusion lasted; and before he became so convinced he could not have been pleased that the Prussians were coming - even though in fact they might have been already approaching (45-6).

In such cases, like LS, we are talking about a person’s attitude toward the sense of a sentence.
When we say that Lois Lane believes that p, the object of her belief is a that-clause, which we have already seen is the name of a proposition, the sense of a sentence.
That-clauses are thus names of things, of senses of the sentences that follow them.

The case of an abstract noun clause, introduced by ‘that,’ includes the case of indirect quotation, in which we have seen the words to have their indirect reference coinciding with what is customarily their sense. In this case, then, the subordinate clause has for its reference a thought, not a truth value... (44-5).

The referents of the terms within that-clauses are similarly indirect.
Aristotle knew of the morning star, so AP is plausible.

AP Aristotle believed that Phosphorus came out in the morning.

Since Hesperus is Phosphorus, Leibniz’s law should allow us to conclude AH from AP.
AH  Aristotle believed that Hesperus came out in the morning.

But AH is, presumably, false, since Aristotle did not know that Hesperus is Phosphorus. Aristotle believed that Phosphorus came out in the morning only under a particular mode of presentation, the sense of the sentence.

In ordinary, direct discourse, we take ‘Hesperus’ and ‘Phosphorus’ to refer to the object Venus. But, in indirect discourse, like the opaque contexts in AP and AH, we must take singular terms to refer to their ordinary senses. In AP and AH, ‘Hesperus’ and ‘Phosphorus’ refer to the object under its mode of presentation for Aristotle. Thus, again, to make sense of sentences like AP and AH, Frege believes that we are forced to posit a distinction between sense and reference.

VII. Two Problems, Briefly

First, here is a problem for modes of presentation, from Stephen Schiffer. The concept that was instantiated by Marina in MK included a mode of presentation. This was important for Frege because it avoided the problems of cognitive content, the Superman-Clark Kent problem and the Aristotle-Hesperus-Phosphorus problem. But if I say, ‘Marina’s owner believed that Marina is a kitten’, I am making a presumably true statement, even though I have no access to the mode of presentation by which her owner knows that Marina is a kitten.

Second, Bob Hale shows that there is a problem with Frege’s analysis of opaque contexts.

BH  My copy of the Grundlagen was on my desk but I thought that it was in my bag.

Since the ‘it’ in BH occurs in an opaque context, it should refer to its sense, not to its ordinary reference. But it refers to the same thing that the term at the beginning of the sentence refers to. That is, ‘it’ is just ‘my copy of the Grundlagen’, not that object under a mode of presentation. So, not all opaque contexts shift references of singular terms to their ordinary senses.

VIII. Ways Not To Be Frege

Not all philosophers adopt Frege’s embrace of senses. Some worry about access to abstract objects. Others just resist the multiplication of entities inherent in Frege’s reification of meanings.

Frege’s sense/reference distinction solves three puzzles. The problem of cognitive content is solved by distinguishing the meaningfulness of names from the identities of their bearers. The problem of empty reference is solved by showing that a sentence containing an empty name can be meaningful when the name has a sense. The problem of opaque contexts is solved by showing that reference within opaque contexts can be to the ordinary senses of names and subordinate clauses.
Any subsequent philosopher who rejects senses must show how to solve these problems. Russell, for example, attempts to solve the problems by using analysis and a description theory. Extensionalists take the meanings of subjects to be just their referents, and the meanings of predicates to be the sets of objects of which those predicates hold. One extensionalist, Quine, solves Frege’s problem with opaque contexts by refusing to substitute into opaque contexts. But, the problems of cognitive content and empty reference are more difficult for extensionalists, as we will see.