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Class 18 - Modalities

Continuing from Tuesday...

I. Identity, rigid designation, and the identity theory

According to Kripke, the identification of water with H_2O, or lightning with electrical discharge, is necessary, just like the identity of heat with molecular motion.
There is a contingent fact about how we experience heat, or lightning, or water.
We pick out heat, or light, according to contingent facts about how they effect us.
But, all theoretical identity statements are, in fact, necessary identities, not contingent identities, p 8.
Since theoretical identity statements are necessary, the identification of pain states with brain states must also be necessary.
For, Kripke claims that pain is a rigid designator, p 11.
Nothing could be a pain if it did not hurt in the way that pains do.
Similarly, if 's' designates a brain state, it does so rigidly.
The identity of any two rigid designators must be necessary, since neither term could refer to anything other than its referent.

There were three (overlapping) considerations against the necessary identification of brain states and mental states.
The first two claimed that if the identity were necessary, then the meanings and the logic of statements involving the terms should be the same.
We can ignore those claims.
Really, the problem distills to this: it seems possible that pain could be something other than a particular state of the brain.
But, if it is possible that pain is not a state of the brain, then the identity of the two must be contingent.
As we have seen, such theoretical identifications must be necessary.
So, the necessary identification must be false.

Thus, Kripke's argument is:

1. The identification of mental states and brain states must be either contingent or necessary.
2. Since mental states and brain states refer rigidly, the identification can not be contingent.
3. Since it is possible that mental states are not states of the brain, the identification can not be necessary.

Thus, mental states and brain states must not be identical.

A different version of the argument relies on the incoherence of contingent identity:

1. Theoretical identities must be necessary.
2. But, it is possible that mental states are not states of the brain.

So, mental states and brain states must not be identical.

II. Kripke and intuitions

The identity theorist agrees with Kripke that the identifications of brain states with mental states should not be necessary.
Kripke's argument against contingent identity depends, in part, on his notion of rigid designation, and its application in this particular case.
Kripke's critic might deny his allegation that 'water', 'lightning', and 'heat' are rigid designators.
If 'water', 'lightning', and 'heat' are not rigid designators, then maybe 'pain' is not rigid either.

His argument seems to rely, once again, on intuitions about what we would say in other possible worlds.
Consider also Kripke's description of the Martians, whose sensations of heat and cold are the reverse of ours.
Kripke claims that we would say that heat exists, even if there are no people to feel it.
If we say, instead, that fires do not heat up the air, but that they transfer molecular motion only, then Kripke's claim that heat rigidly designates is unsupported.
Note that this criticism of Kripke's claim uses the old primary-secondary distinction.
According to Locke, objects do not have color, or heat, or taste in them.
These sensations, or secondary properties, arise only from our interactions with objects which, technically, have only the properties used in physics, the primary qualities.
If we accept the primary/secondary distinction, then Kripke's claim that heat exists, even if there are no people to feel it, is just false.
On the Lockean view, there is just heat, as we feel it, and molecular motion.

What do we say about possible worlds?
What do we say about Twin Earth?
What do we say about zombies, and homunculi-headed robots?
Moreover, why do we say what we say?
Kripke's claims are based on intuitions about possible worlds.

Kripke, the philosopher, started his career as Kripke, the modal logician.
In *Naming and Necessity*, he argues that we stipulate other possible worlds.
The language of possible worlds just represents the way in which we have come to talk about counter-factual possibilities, like the possibilities of zombies, or homunculi-headed robots.
Still, we need a base from which to argue about them.
We need some account of the basis on which we make claims about other possible worlds.

III. Possible worlds

Kripke's argument against the identity theory depended on intuitions about possible worlds, both in order to make sense of rigid designation, and in order to make specific claims about pain and brain states.
Furthermore, almost all of the arguments we have discussed this term in the philosophy of mind rely, in some way, on our intuitions about possibility.
Our concern, at this point, is not so much with Kripke's argument against identity theory.
The readings in the second half of this course will, in places, not only talk about counterfactuals in the casual way in which, say, Ned Block talks about them.
Some readings will also presuppose a bit of familiarity with the mechanics of modal logic, which is the logicians' tool for discussing possible worlds.

So, we are looking at Kripke's methodology.
Two questions that his methods raise are:

1. What are possible worlds?
2. How do we know about possible worlds?

The first question regards the metaphysics of possible worlds.
The second question regards the epistemology of possible worlds.
As always, these questions are related.
Any account of the existence of possible worlds will demand an account of how we know about them.
Any denial that we can know about possible worlds will lead to a demand for an account of our ability to make significant counterfactual claims.

Nolt characterizes possible worlds as the result of extending from possible situations to entire states of the universe.
Leibniz conceived states of the universe as having complete histories, and complete futures.
He was a realist about possible worlds, in that he believed that God could comprehend all of them existing at the same time.
There are two distinct ways of being modal realists.
We can think of possible worlds as concrete, or as abstract.

David Lewis thinks of possible worlds as concrete objects, all co-instantiated, all existing.
He argues that there is no reason to prefer any possible world, metaphysically, to any other possible world.
Thus, he believes that all possible worlds exist, in the same way that our world exists.
You might look at his *On the Plurality of Worlds*.

Slightly less contentiously, we might, with Robert Stalnaker, consider possible worlds as abstract objects, like numbers and propositions.
See his collection *Ways the World Might Be*.
Note that Kripke says, at fn 15, that the phrase 'possible worlds' is misleading.
Instead, we should think of them as possible states of this world, which is how Stalnaker thinks of them.
Since they are not actual states of the world, it is a bit of a puzzle what to make of them.
Stalnaker understands worlds to be sets of propositions.
The world is just the set of propositions that are true.
So, a possible world is just a different set of propositions.
Actually, Stalnaker takes propositions to be sets of possible worlds, the set of all worlds at which a proposition is true.
He thus takes possible worlds as brute, and defines propositions in terms of them.
Both ways, the connection between propositions and possible worlds, and the identification of possible worlds as abstract objects, holds.]

In contrast to modal realism is the claim that the only things that exist are actual objects in the actual world, and that all talk of modality is dispensable, or nonsensical.
This view may be called actualism, or modal anti-realism.
Quine, for example, thinks that all talk of modal logic is faulty, and that there are no possible worlds.
You might look at his "Reference and Modality" in *From a Logical Point of View*, and "Three Grades of Modal Involvement" in *The Ways of Paradox*.

Actualism is motivated mainly by epistemic worries about realism.
How could we possibly know anything about other possible worlds when all of our experience is with this world?
If there were necessary truths, then perhaps we could know those.
That is, we might argue that we know that 7+5=12 in all possible worlds.
But, claims about contingent facts in other possible worlds seem unknowable, since we lack any access to them.
Modal actualism is epistemically sterile.
But, the modal actualist can not say anything about possibility.
Experience, as Hume taught us, only tells us how things were.
It is silent on how things must be.

Most modal realists, and people who take the actualist's epistemic worries seriously, look for some plausible story about how we know about possible worlds.
One way to avoid problems about access to other possible worlds is to claim, with Kripke's fn 15, that we stipulate other possible worlds.
Still we stipulate some kinds of facts, and avoid others.
We stipulate that I could have been a psychologist.
We do not stipulate that I could have been a trash can.
We stipulate that the shirt I am wearing could be another color.
We do not stipulate that it could lack color altogether.

One way to alleviate epistemic worries is to claim that everything that we know about possible worlds is based, or supervenes, on our knowledge of the actual world.

Supervenience: Modal facts supervene on non-modal facts.

A set of properties or facts M supervenes on a set of properties or facts P if and only if there can be no changes or differences in M without there being changes or differences in P.
For example, one might think that mental properties supervene on physical properties, even if they can not be reduced to physical properties.
Supervenience entails that there are no facts that are non-modal.
All our knowledge of possibility comes from our knowledge of the actual world.

The term 'possible worlds' might make the problem of access seem more difficult than it is.
We do seem to know that certain events are possible and others are not.
If we do not want to posit possible worlds, and we can not ground our knowledge of possibility in our experience of this world , we might argue that we know about them from pure reason.
Pure reason, though, seems to many people like another word for intuition.
And so we are back to the worries about intuition that started us thinking about these questions.

What follows is drawn from the lecture notes I prepared for my logic class on modal logic.
(I tweaked those notes a bit.)
It will be important to have a sense of the modal operators, and a bit of how the reasoning works.
The technical details I present are simpler than Nolt's presentation, since they only concern propositional modal logic, rather than quantificational modal logic.

IV. Modal operators

Consider the following sentences:

A. It is not the case that the sun is shining.
B. It is possible that the sun is shining.
C. It is necessary that the sun is shining.

Sentence A contains a sentential operator, negation.
Negation is a function that takes truth values to their opposites.
Sentences B and C contain operators, as well.
These operators are kinds of modal operators, and the logic of these kinds of operators is called modal logic.
The formation rules for propositional modal logic are the same as those for propositional logic, with one exception:

Formation rules for propositional modal logic

1. A single capital English letter is a wff.
2. If α is a wff, so is $\sim\alpha$.
3. If α and β are wffs, then so are $(\alpha \cdot \beta)$, $(\alpha \vee \beta)$, $(\alpha \supset \beta)$, and $(\alpha \equiv \beta)$.
4. If α is a wff, then so is $\Diamond\alpha$.
5. These are the only ways to make wffs.

Modal logic is actually the study of a family of related phenomena.
I will introduce a bit of formal modal logic, and stress the primary interpretation (in terms of possibility and necessity) of the modal logic symbols.
The modal logic of possibility and necessity is called alethic logic, but there are several different interpretations of the modal operators.

V. Alethic operators

In alethic modal logic, the '$\Diamond$' is interpreted as 'it is possible that'.
If we take 'S' to stand for 'the sun is shining', then sentence B is regimented as '$\Diamond S$'.
So, '$\Diamond\sim S$' means that it is possible that the sun is not shining.
And, '$\sim\Diamond S$' means that it is not possible that the sun is shining.
And, '$\sim\Diamond\sim S$' means that it is not possible that the sun is not shining.
That last sentence is, you may notice, equivalent to sentence C, above.
We introduce another operator, '$\Box$', for necessity, with the following stipulation:

$$\Box\alpha :: \sim\Diamond\sim\alpha$$

Similarly:

$$\Diamond\alpha :: \sim\Box\sim\alpha$$

That is, only one modal operator is normally taken as basic, and the other is introduced by definition.

Studies of alethic operators trace back to Leibniz's interest in possible worlds.
Leibniz thought that we lived in the best of all possible worlds, and that this fact followed from the power and goodness of God.

1. God is omnipotent so he can create the best possible world.
2. God is omni-benevolent, so he wants to create the best possible world.
3. The world exists.

So, this is the best of all possible worlds.
Corollary: All of the evil in this world is necessary.

See Voltaire's wonderful satire of Leibniz's views in *Candide*.

Kripke's contribution to the modal logic literature was the development of possible world semantics.
Semantics is the study of interpretations of formal systems.
Semantics are most simply presented using truth tables.
Kripke semantics uses the language of possible worlds to interpret sentences of modal logic.

VI. Actual World semantics

$V(\sim\alpha) = \top$ if $V(\alpha) = \bot$; otherwise $V(\sim\alpha) = \bot$
$V(\alpha \bullet \beta) = \top$ if $V(\alpha) = \top$ and $V(\beta) = \top$; otherwise $V(\alpha \bullet \beta) = \bot$
$V(\alpha \vee \beta) = \top$ if $V(\alpha) = \top$ or $V(\beta) = \top$; otherwise $V(\alpha \bullet \beta) = \bot$
$V(\alpha \supset \beta) = \top$ if $V(\alpha) = \bot$ or $V(\beta) = \top$; otherwise $V(\alpha \supset \beta) = \bot$
$V(\alpha \equiv \beta) = \top$ if $V(\alpha) = V(\beta)$; otherwise $V(\alpha \equiv \beta) = \bot$

Consider the following propositions:

P: The penguin is on the TV.
Q: The cat is on the mat.
R: The rat is in the hat.
S: The seal is in the sea.

Suppose that in our world, call it w_1, P, Q, R, and S are all true.
We can easily translate the following claims, and determine their truth values.

Exercises A: Translate each of the following claims into English, and determine their truth values, given the translation key and truth assignments above.

1. $P \supset Q$
2. $P \supset R$
3. $\sim(R \vee S)$
4. $(P \bullet Q) \supset R$
5. $(Q \vee \sim R) \supset \sim P$

IV. Possible World semantics (Leibnizian)

Consider another world, call it w_2, in which P and Q are true, but R and S are false.
That is, in w_2, the penguin is on the TV and the cat is on the mat, but the rat is not in the hat and the seal is not in the sea.
To represent the differences between w_1 and w_2, we need to extend actual world semantics to possible world semantics.
In actual world semantics, valuations are one-place functions.
In possible world semantics, valuations are two-place functions, of the proposition and the world at which we are considering the proposition.
At each world, the valuations are just as in the actual world.
But, by indexing our formulas, we can consider statements whose truth values depend on the facts in other possible worlds.
First, we consider a universe, U, which is just a set of worlds: $\{w_1, w_2, w_3,...,w_n\}$
Then, we introduce the indexed rules.

$V(\sim\alpha, w_n) = \top$ if $V(\alpha, w_n) = \bot$; otherwise $V(\sim\alpha, w_n) = \bot$
$V(\alpha \bullet \beta, w_n) = \top$ if $V(\alpha, w_n) = \top$ and $V(\beta, w_n) = \top$; otherwise $V(\alpha \bullet \beta, w_n) = \bot$
$V(\alpha \vee \beta, w_n) = \top$ if $V(\alpha, w_n) = \top$ or $V(\beta, w_n) = \top$; otherwise $V(\alpha \vee \beta) = \bot$
$V(\alpha \supset \beta, w_n) = \top$ if $V(\alpha, w_n) = \bot$ or $V(\beta, w_n) = \top$; otherwise $V(\alpha \supset \beta, w_n) = \bot$
$V(\alpha \equiv \beta, w_n) = \top$ if $V(\alpha, w_n) = V(\beta, w_n)$; otherwise $V(\alpha \equiv \beta, w_n) = \bot$

Let's consider a small universe:

U= $\{w_1, w_2, w_3\}$
At w_1, P, Q, R and S are all true.
At w_2, P and Q are true, but R and S are false.
At w_3, P is true, and Q, R, and S are false.

Now, let's return to the sentences from Exercises A.
At w_1, their values remain as above.
We can also evaluate them for w_2 and w_3.

Exercises B: Determine the truth values of each of the following claims at w_2 and w_3.

1. $P \supset Q$
2. $P \supset R$
3. $\sim(R \vee S)$
4. $(P \bullet Q) \supset R$
5. $(Q \vee \sim R) \supset \sim P$

For ease of identification, we can index propositions.
So, we can use 'P_1' to mean that the penguin is on the TV at w_1.

$P_1 \supset P_3$	means that if the penguin is on the TV in w_1, then it is on the TV in w_3
$\sim(Q_2 \bullet Q_3)$	means that it is not the case that the cat is on the mat in both w_2 and w_3

Exercises C: Translate each of the following claims into English, and determine their truth values

1. $P_1 \supset (P_2 \bullet P_3)$
2. $S_1 \bullet S_2 \bullet S_3$
3. $R_1 \vee R_2 \vee R_3$
4. $[(P_1 \vee P_2) \supset (Q_1 \vee Q_2)] \bullet (P_3 \supset \sim Q_3)$

Until now, we have looked at non-modal propositions in other possible worlds.
What about modal claims, like '$\Diamond S$'?
Here is a Leibnizian semantics for the modal operators:

$V(\Box\alpha, w_n) = \top$ if $V(\alpha, w_n) = \top$ for all w_n in U
$V(\Box\alpha, w_n) = \bot$ if $V(\alpha, w_n) = \bot$ for any w_n in U
$V(\Diamond\alpha, w_n) = \top$ if $V(\alpha, w_n) = \top$ for any w_n in U
$V(\Diamond\alpha, w_n) = \bot$ if $V(\alpha, w_n) = \bot$ for all w_n in U

So, '$\Diamond S_1$' is true, since there is at least one world, w_1, in which the seal is in the sea.
Similarly, '$\Box Q$' will be false, since there is at least one world, w_3, in which the cat is not on the mat.

Exercises D: Determine the truth values of each of the following propositions, given the semantics and assignments above.

1. $\Box(P \supset Q)_1$
2. $\Diamond(P \supset Q)_1$
3. $\Box P_1 \supset \Box Q_1$
4. $\Diamond P_1 \supset \Diamond Q_1$
5. $\Diamond[(Q \vee \sim R) \supset \sim P]_1$
6. $\Diamond P_1 \supset [Q_1 \supset \Box(R \bullet S)_1]$
7. Which of the truth values of the above sentences vary if considered at w_2 or w_3 (i.e. if we replace all the subscripts with '2's or '3's)?

V. Possible world semantics (Kripkean)

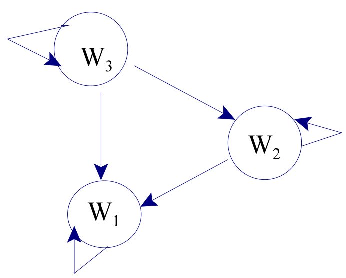
Leibnizian semantics suffices to express logical possibility.
But, there are more kinds of possibility than logical possibility.
For example, it is logically possible for a bachelor to be married, for objects to travel faster than the speed of light, and for a square to have five sides.
These claims are logically possible, even though they might be semantically impossible, or physically impossible, or mathematically impossible.
So, a more subtle version of modal logic might be useful.

Consider two possible worlds, as Nolt does, p 336.
The first world is ours.
The second world is like ours, except that there is some force which moves the planets in perfectly circular orbits.

Now, the law that says that all planets move in elliptical orbits holds in both worlds, since a circle is just a type of ellipse.
So, w_2 obeys all the laws of w_1, but w_1 does not obey all the worlds of w_2.
The modal logician describes this difference in terms of accessibility.
w_2 is accessible from w_1, but w_1 is not accessible from w_2.
Thus, for Kripkean modal logics, in addition to different possible worlds, we need an accessibility relation.
Accessibility relations entail that propositions that are possible at some worlds are not possible at other worlds.

Here is one possible accessibility relation that we can stipulate among our worlds w_1, w_2, and w_3.

$$R = \{<w_1, w_1>, <w_2, w_1>, <w_2, w_2>, <w_3, w_1>, <w_3, w_2>, <w_3, w_3>\}$$



Accessibility relations can be given diagrammatically, too.

Now, we extend our Leibnizian semantics into a Kripkean semantics:

$V(\Box\alpha, w_n) = \top$ if $V(\alpha, w_m) = \top$ for all w_m in U such that $<w_n, w_m>$ is in R
$V(\Box\alpha, w_n) = \bot$ if $V(\alpha, w_m) = \bot$ for any w_m in U such that $<w_n, w_m>$ is in R
$V(\Diamond\alpha, w_n) = \top$ if $V(\alpha, w_m) = \top$ for any w_m in U such that $<w_n, w_m>$ is in R
$V(\Diamond\alpha, w_n) = \bot$ if $V(\alpha, w_m) = \bot$ for all w_m in U such that $<w_n, w_m>$ is in R

The introduction of accessibility relations turns modal logic into an enormous field.
We can characterize different kinds of relations.
For example, an accessibility relation might be an equivalence relation: reflexive, symmetrical, and transitive.
A relation is reflexive if every object bears the relation to itself.
In the case above, every world is accessible from itself.
A relation is symmetric if a bearing R to b entails that b bears R to a.
The relation above is not symmetric since w_2 is accessible from w_1, but w_1 is not accessible from w_2.
A relation is transitive if given that a bears R to b and b bears R to c, it follows that a bears R to c.
The case above is transitive.
If the accessibility relation is an equivalence relation, the universe will be Leibnizian.
But, there are much more restrictive kinds of accessibility relations.

Let's go back to our located animals, and see how they fare in a Kripkean universe with the given accessibility relation.

> U= $\{w_1, w_2, w_3\}$
> At w_1, P, Q, R and S are all true.
> At w_2, P and Q are true, but R and S are false.
> At w_3, P is true, and Q, R, and S are false.
> R= $\{<w_1, w_1>, <w_2, w_1>, <w_2, w_2>, <w_3, w_1>, <w_3, w_2>, <w_3, w_3>\}$

Exercises E: Determine the truth values of each of the following formulas.

1. $\Box(P \supset Q)_1$
2. $\Box(P \supset Q)_3$
3. $\Diamond{\sim}(Q \vee R)_1$
4. $\Diamond{\sim}(Q \vee R)_2$
5. $\Diamond{\sim}(Q \vee R)_3$
6. $\Box P_1 \supset \Box Q_1$
7. $\Box P_3 \supset \Box Q_3$

VI. Various other modal systems, and their characteristic axioms

Different accessibility relations determine different modal systems.
Different modal systems support different modal intuitions.
For example, the alethic interpretation demands a reflexive relation.
If an accessibility relation is not reflexive, then it would be possible for '$\Box$P' to be true in the same world in which 'P' were false.
But if P is necessarily true, then it surely should be true.
On the other hand, a non-reflexive relation supports a deontic interpretation of the modal operators, which we will discuss, below.

Modal systems can be characterized both by their accessibility relations and by the modal claims they generate.
For instance, modal system K has the following characteristic axiom.

> K: $\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$

We can construct an axiomatic derivation system based on K.
System K has the axiom above, as well as the rules of modus ponens, and necessitation:

> (Nec) α / $\Box\,\alpha$

Necessitation looks a little odd, until you recognize that the only things you prove, without premises, are logical truths.
We can also take as a rule that any tautology in propositional logic is a theorem (Thm).

We can derive another rule, called regularity

(Reg) α ⊃ β / □α ⊃ □β

We will show that Reg is valid by deriving '□P ⊃ □Q' from an arbitrary 'P ⊃ Q'

1. P ⊃ Q / □P ⊃ □Q
2. □(P ⊃ Q) 1, Nec
3. □(P ⊃ Q) ⊃ (□P ⊃ □Q) Axiom K
4. □P ⊃ □Q 3, 2, MP

QED

Now, let's do a slightly more complicated proof, of the modal claim: □(P • Q) ⊃ (□P • □Q)

1. (P • Q) ⊃ P Thm
2. □(P • Q) ⊃ □P 1, Reg
3. (P • Q) ⊃ Q Thm
4. □(P • Q) ⊃ □Q 3, Reg
5. [□(P • Q) ⊃ □P] ⊃ {[□(P • Q) ⊃ □Q] ⊃ [□(P • Q) ⊃(□P • □Q)]} Thm
6. [□(P • Q) ⊃ □Q] ⊃ [□(P • Q) ⊃(□P • □Q)] 5, 2, MP
7. □(P • Q) ⊃(□P • □Q) 6, 4, MP

QED

Note that the theorem used on line 5 has a simple instance: '(P ⊃ Q) ⊃ {(P ⊃ R) ⊃ [P ⊃ (Q • R)]}'

K is a very weak modal logic.
A slightly stronger logic, D, has the characteristic axiom

D: □α ⊃ ◊α

Every theorem provable in K is provable in D, but D allows for more to be proven.
Thus, it is a stronger logic, and more contentious.
Let's consider the meaning of the characteristic axiom of D.
In alethic interpretation, it means that if a statement is necessary, then it is possible.
This seems reasonable, since necessary statements are all true.
And, true statements are clearly possible.
But, an interesting fact about D is that the following theorem is not provable:

T: □α ⊃ α

Thus, D seems like a poor logic for the alethic interpretation.
But, there are other interpretations of the modal operators, like the deontic one.
In deontic logic, '□P' means that P is obligatory, and '◊P' means that P is permissible.
Now, the characteristic axiom of D seems true under this interpretation.
I must be permitted to perform any action that I am obliged to do.
Further, T also seems true, since from the fact that an action is obligatory it does not follow that people actually do it.

Another interpretation of the modal logic symbols leads to epistemic logic.
For epistemic logic, we take ‘□P’ to mean that P is known, and ‘◊P’ to mean that P is compatible with things that are known.
Hintikka’s epistemic logic takes three axioms:

K: $\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta)$
T: $\Box\alpha \supset \alpha$
4: $\Box\alpha \supset \Box\Box\alpha$

Any logic with the T axiom will have a reflexive accessibility relation.
Any logic with the 4 axiom will also have a transitive accessibility relation.
Note that 4 is contentious in epistemic logic.

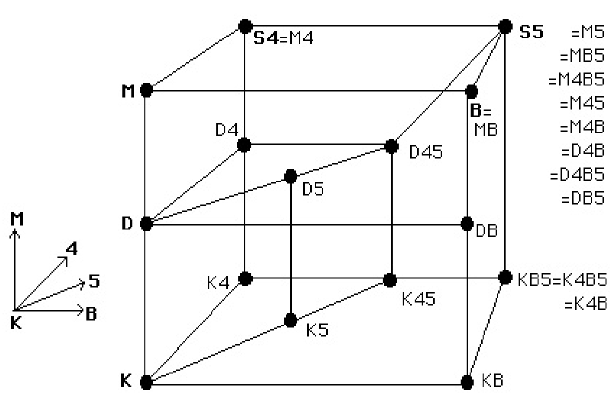
A system with K, T, and 4 is called S4.
One can get a slightly stronger logic, by adding a condition of symmetry to the accessibility relation.
Then, we have S5, which takes K, T, 4, and:

B: $\alpha \supset \Box\Diamond\alpha$

In S5, the accessibility relation is an equivalence relation.
S5 is often seen as the best alethic system, though it best captures logical necessity rather than other kinds.

There are other interpretations of the modal operators.
A slight tweak of epistemic logic is the logic of belief.
More interestingly, we can generate temporal logics, in which ‘□P’ means that P is always the case, and ‘◊P’ means that P is the case at some point in time.
Temporal logics can be extended to include tense operators, for the future and the past.

One last interpretation of the modal operators has become especially important in contemporary philosophy of science and mathematics.
Hartry Field uses the modal operator to represent consistency.
On Field’s view, ‘◊(P • Q • R)’ means that P, Q, and R are consistent.
Normally, consistency is a metalogical notion, discussed in a metalanguage.
Field argues that the consistency of a set of sentences is actually a logical notion.
Thus, we need to have symbols in the language to represent it.



This diagram represents the relations between the different modal logical systems and their axioms.

Garson, James, "Modal Logic", The Stanford Encyclopedia of Philosophy (Fall 2008 Edition), Edward N. Zalta (ed.), URL = <http://plato.stanford.edu/archives/fall2008/entries/logic-modal/>.

VII. A criticism of modal logic, from Quine

Consider the following two claims.

V: The number of planets is greater than seven.
W: Eight is greater than seven.

V and W have the same truth value.
We expect that they will have the same truth value, since one can be generated from the other by a simple substitution, given that:

X: The number of planets is eight.

Now, make V and W modal claims.

Y: Necessarily, the number of planets is greater than seven.
Z: Necessarily, nine is greater than seven.

Now, Y and Z have different truth values.
But, we can still generate one from the other, with the given substitution.
Quine's criticism of modal logic has been influential; again see "Reference and Modality".

VIII. Solutions

Answers to Exercises A:
1. If the penguin is on the TV, then the cat is on the mat; true
2. If the penguin is on the TV, then the rat is in the hat; true
3. Neither the rat is in the hat nor the seal is in the sea; false
4. If the penguin is on the TV and the cat is on the mat, then the rat is in the hat; true
5. If either the cat is on the mat or the rat is not in the hat, then the penguin is not on the TV; false

Answers to Exercises B:
1. True at w_2, false at w_3
2. False at w_2 and w_3
3. True at w_2 and w_3
4. False at w_2, true at w_3
5. False at w_2 and w_3

Answers to Exercises C
1. True
2. False
3. True
4. True

Answers to Exercises D:
1. False
2. True
3. False
4. True
5. False
6. False
7. #6 is true at w_3

Answers to Exercises E:
1. True
2. False
3. False
4. False
5. True
6. True
7. False